The Enduring Effects of Unconventional Monetary Policy *

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Abstract

Does unconventional monetary policy generate a long-term impact on macroeconomic outcomes? This paper investigates the transmission in an economy where credit-financed R&D investment drives endogenous growth. Using a dynamic general equilibrium framework that links financial conditions and growth, I study the long-run aggregate effects of quantitative easing (QE), forward guidance (FG) and negative interest rate policy (NIRP). All expansionary measures operate through the credit channel, improving banks' balance sheet conditions and fostering economic growth. In calibrated scenarios, FG and NIRP emerge as the most effective tools for sustaining productivity increases. These policies boost TFP and output, mitigating the ZLB constraint. While QE raises TFP persistently, its quantitative impact is smaller, with a more short-lived effect on output.

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JEL Classification: *E22, E24, E32, E44, E52, G01.*

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1 Introduction

In the aftermath of the Global Financial Crisis, central banks around the world undertook unprecedented measures to stabilize economies and prevent a descent into prolonged recession. Conventional monetary policy tools, such as interest rate adjustments, were limited downward by the ZLB constraint and revealed to be insufficient in the face of the severe economic challenge. This realization prompted central banks to explore uncharted territories, giving rise to what has come to be known as unconventional monetary policy (UMP). The broadly defined set of unconventional monetary policies can be further and better subdivided among three instruments that can, or have been, effectively used to counteract the meltdown arising from financial and sovereign debt panics. The first attempt has gone under the name of quantitative easing (QE) and consisted in massive liquidity injections performed by central banks through the purchase of private and government assets, aiming to restore the functioning of the lending sector and put downward pressure on long-term interest rates, in times of credit crunches and financial disruptions. In the US, it comprised the *Large-Scale Asset Purchases* programs (*LSAPs*, also known as rounds of QE1, QE2, QE3 and QE4), and the *Maturity Extension Program (MEP)*.[1] To offer a glimpse about the magnitude of the interventions, figure 1 reports the *Federal Reserve* balance sheet dynamics, from which it is visible the massive asset expansion inaugurated from 2008 onwards. At the same time, a further implemented alternative consisted in the central bank releasing public communications about the likely path of its conventional policy, i.e. future movements of the policy interest rates. This measure is defined Forward Guidance (FG), where information are released as an attempt to influence prices and interest rates through the expectation channel. In particular, the guidance might or might not denote a commitment of the central bank to future actions, distinguishing between the FG *"Odyssean"* and *"Delphic"* components. Figure 2 represents investors' expectations for the future behaviour of the Fed Funds Rate, where a longer persistence about the interest rate liftoff emerges after the *Federal Reserve* started FG communications. Finally, a long debated option was represented by exceeding the ZLB threshold and setting the policy

[1] It is generally useful to distinguish among several QE typologies, avoiding to comprehend all possible policy ways under the same definition. However, this distinction overcomes the objectives of the current analysis. A thorough review of the topic is offered in Kuttner (2018).

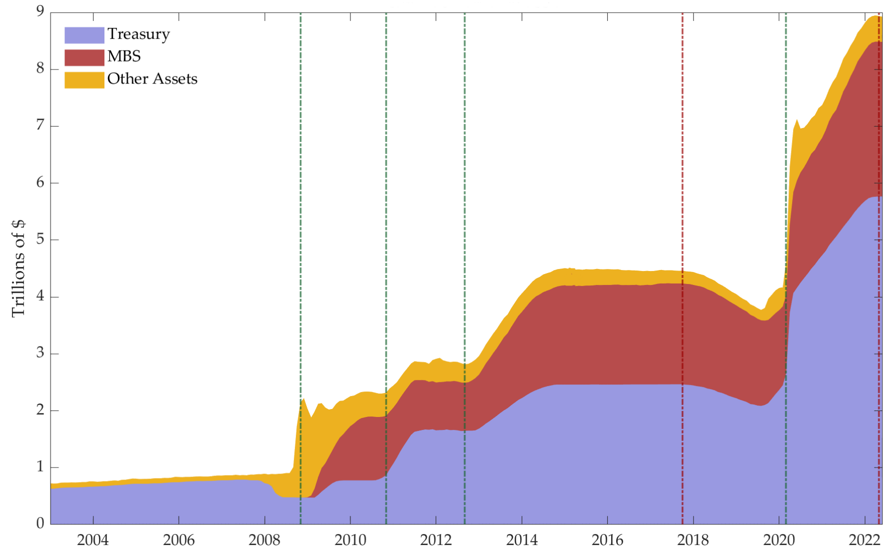

Figure 1: *Federal Reserve* Balance Sheet

Balance Sheet by Asset Composition, 2003.1 - 2022.6, Trillions of US $: Treasury Securities (purple area), Mortgage Backed Securities (red area), Other Assets (yellow area). Green vertical lines represent the start of QE programs, red vertical lines the start of QT. Source: *Federal Reserve Bank of Richmond*.

rate below zero, i.e. applying a negative interest rate policy (NIRP). Purpose of the measure lies in the will to boost private sector consumption and investment expenditures, as the policy imposes a cost over the opportunity to hold savings. On the other side, it has the potential to shrink financial intermediaries' margins, augmenting the drying up of the bank lending market. Accordingly, the existence of an Effective Lower Bound (ELB) may constitute a real limit to this option, over which the soundness of the banking system may not be guaranteed. Figure 3 shows the experience of the European and Swedish Central Banks implementing negative deposit rates.

The efficacy of unconventional monetary policy, characterized by initiatives such as QE, FG and NIRP, has been a subject of intense academic and policy debate. Beyond the mixed evidence offered by scholars about the ability of such measures in averting immediate economic collapse, I here go one step further and question the long-term consequences. As economies gradually recover, it becomes imperative to assess and understand the enduring impact of UMP on various facets of the financial system, economic structure, and the behavior of key economic agents.

Therefore, this paper has the purpose of delineating the theoretical mechanisms below the long-run functioning of unconventional monetary policies. The research in Sims &

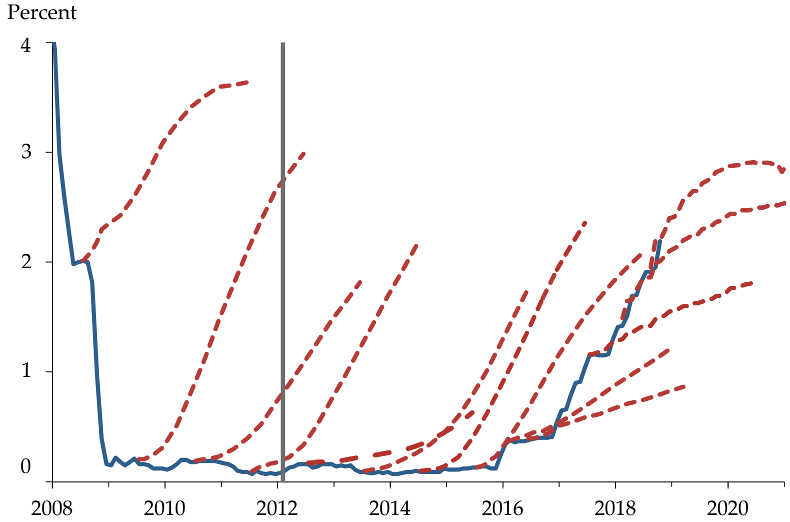

Figure 2: *Federal Reserve* Forward Guidance

Dynamics of the actual Fed Funds Rate (blue, solid) and financial investors' midyear expectations for the path of the future Funds Rate (red, dashed), 2008.Q1 - 2020.Q4. Grey vertical line denotes the introduction of forward guidance communications (2012.Q1). Source: Rudebusch (2018).

Wu (2021) represents the milestone of the following analyses. Their work establishes a general framework allowing to jointly study the three UMP categories mentioned above. Few elements, added to the structure of a basic medium-scale DSGE model, permit to perform it. The first key is the presence of a banking sector, where intermediaries collect resources under the form of deposits (i.e. short-term debt) and hold long-term private and government bonds, in addition to reserves emitted by the central bank. This representation relies upon the sketch of a financial market populated by perpetual bonds, represented under the form of zero coupon long-term bonds, issued by production firms as an instrument to finance investment. Markets are segmented, such that the availability of long-term bonds is not at the households' disposal. The above features, together with a usual costly enforcement constraint that limits the intermediaries activity and creates space for excess returns, introduce an *"investment wedge"*, which is the channel linking UMP measures with the real economy and determining their effect on macro aggregates. In addition, the central bank self-finances through the issuance of interest-bearing reserves, while the same authority imposes a binding requirement on the amount of reserves a private bank is allowed to hold. This modelling represents the shortcut to successfully study the potential impact of a NIRP implementation, in case the ZLB constrains the deposit interest rate. Finally, the

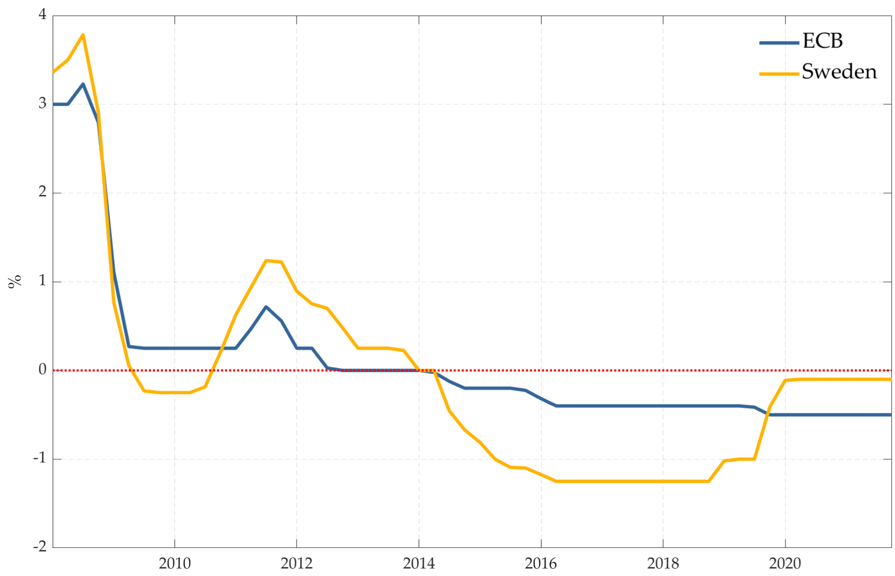

Figure 3: Negative Interest Rate Policy

Interest rates on deposits applied by the *European Central Bank* (*"ECB Deposit Facility Rate"*, solid blue) and the *Sveriges Riksbank* (*"Key Deposit Rate"*, solid yellow), 2008.Q1 - 2021.Q4, quarterly average, percentage.

study of forward guidance benefits from an innovative way of introducing the effect of central bank communications, embodied by a current shock to the desired policy rate instead of the announcement of an equivalent shock in the future.

How do UMP influence economies in the long run? In order to address the initial research question, the above world is augmented by frictions that endogenize growth, through a dedicated sector which performs R&D activities. With respect to the strategy presented in Beqiraj et al. (2025), this time the analysis delineates a simpler version of the innovation process, based on the work of Queralto (2020), that abstracts from distinguishing between creation and adoption of new technologies.[2] A unique actor, namely the innovator, devotes units of final output to the investment in R&D. According to the horizontal growth paradigm, this effort translates into new output good varieties that represent the endogenous component of total factor productivity, as in Anzoategui et al. (2019). Thus, R&D investments embody the driver of future TFP growth. Moreover, I modify innovators' behaviour assuming they experience a *"loan-in-advance constraint"* akin to wholesale producers, where R&D is limited by a binding

[2]While such a distinction is relevant from both the qualitative and quantitative sides of the analysis, as proved in Beqiraj et al. (2025), it does not alter the bulk of results about the transmission of UMP policies on the long run, which is guaranteed by the presence of an endogenous growth engine.

hurdle that requires the issuance of perpetual bonds to finance innovation expenditure. The central bank can buy bonds issued by the innovation sector, but can also influence its activity through the impact of FG and NIRP transmitting along the credit channel. This framework directly ties UMP implementation to innovation, strengthening the connection with the real economy and explaining what effects these policies determine beyond the business cycle frequency.

The key results can be summarized as follows. First, a conventional expansionary monetary shock increases TFP and output persistently, operating through a decline in credit spreads and a subsequent expansion of bank lending to innovation activities. When the ZLB becomes binding, unconventional tools replicate these mechanisms with different magnitudes and persistence. QE reduces credit spreads and stimulates TFP growth, yet its effects on output are more short-lived, as the liquidity expansion fades once the central bank balance sheet stabilizes. FG emerges as a highly effective policy instrument: despite operating under the ZLB constraint, it successfully anchors expectations of lower future rates, fostering credit expansion, innovation investment and a permanent rise in productivity, thereby reproducing the stabilization outcomes of a conventional rate cut. NIRP also yields an aggregate expansion comparable to FG, combining a signalling channel similar to forward guidance with a direct reduction in reserve rates. However, it may compress bank profitability and tighten balance-sheet constraints, slightly moderating its effectiveness. Overall, the simulations show that FG and NIRP are the most powerful tools for sustaining long-term productivity growth, while QE exerts a smaller but persistent effect on TFP. All expansionary measures operate through the credit channel, improving banks' balance-sheet conditions and enhancing the flow of credit toward R&D investment, i.e. the key driver of endogenous growth.

Literature. As the biggest effort of monetary policy after the GFC and sovereign debt crises was represented by the implementation of unconventional tools, scholars surveyed the effects deriving from this typology of interventions. Clearly from the following works, UMP (particularly under the form of QE) was influential at stimulating corporate investments, including R&D, thus representing a tool to increase TFP and output at longer horizons.

Grimm, Laeven & Popov (2021) are in favour of a long-run impact induced by QE measures. Studying the *European Central Bank* emergency liquidity program, they documented a clear and powerful change on the level of R&D investment performed by firms, consequence of the variation in financing conditions that the *"Corporate Sector Purchasing Program"* determined. They discovered program-eligible firms experienced a higher increase in the R&D effort with respect to comparable but ineligible counterparts, although this result was true only for companies characterized by low indebtedness levels, which previously already were strong innovators. Further interesting results descend from the apparent missing relevance of credit constraints at driving the effect of QE on R&D investment.

Giambona, Matta, Peydró & Wang (2025) highlighted a further channel about the QE positive incentive to business investment: the *"corporate-bond lending channel"*. As the *Federal Reserve*'s QE program performed a large-scale of asset purchases, particularly absorbing Mortgage Backed Securities and Treasuries, the immediate repercussion was a lack of available safe assets. This policy introduced a bias in the market, where safer firms increased their propensity to invest through the issuance of *"safe"* bonds. From a quantitative perspective, authors found asset purchases spurred firm-level investment by more than 7% points (provided these firms were granted of market access), where this effect was supported by newly-released senior bonds.

Swanson (2023), building on his previous research, addresses the question of the persistence arising from a set of unconventional monetary policies. He identifies and estimate the consequences of innovations in the Federal Funds Rate, forward guidance and LSAPs, through high-frequency interest rate variations around major announcements made by the FED Board, which are then embodied as external IV within a structural VAR. While FFR changes are found to provoke the largest impact on the US economic environment, deviations in FG and LSAPs are respectively still relevant but less important in terms of magnitude. In particular, focusing on the two unconventional measures, FG announcements determine persistent movements in interest rates, spreads and output, up to 50 months after a tightening shock. Asset purchases shocks instead create more short-lived and weakest effects with respect to other kinds of policies, results which are broadly shared by Miranda-Agrippino & Ricco (2023) under a similar framework analysis.

Rest of the paper is composed as follows: section 2 describes the theoretical model, section 3 presents the results, section 4 concludes.

2 Model

The model economy includes the following actors: households, labor unions, non-financial firms, financial intermediaries, innovators, the government and a central bank.

2.1 Households

A unit measure continuum of households gets utility from consumption and leisure. Following Queralto (2020), households are composed of workers and bankers and supply both skilled and unskilled labour. They aim to maximize lifetime utility, represented by:

$$\max_{C_t, L_{ut}, L_{st}, D_t} E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{\left(C_{t+j} - h\, C_{t+j-1}\right)^{1-\varrho}}{1-\varrho} - \chi_t^u \frac{\left(L_{ut+j}\right)^{1+\varphi}}{1+\varphi} - \chi_t^s \frac{\left(L_{st+j}\right)^{1+\varphi}}{1+\varphi} \right\} \tag{1}$$

subject to the following budget constraint:

$$P_t C_t + D_t - D_{t-1} \leq W_{ut} L_{ut} + W_{st} L_{st} + DIV_t - P_t H_t - P_t T_t + \left(R^d_{t-1} - 1\right) D_{t-1} \tag{2}$$

D_t represents the nominal value of banks' deposits, R^d_t is the nominal gross interest rate at which deposits are remunerated, W_{ut}, W_{st} the wages for unskilled (L_{ut}) and skilled (L_{st}) labour, χ^u_t, χ^s_t denote the disutilities attached to labour, h shows habit in consumption, DIV_t are the incoming dividends from owning banks and non-financial firms, H_t is a transfer devoted to new bankers under the form of net worth, T_t are lump-sum taxes.[3]

[3]Labour disutilities are defined by the following functional forms: $\chi^u_t = \chi^u A_t^{1-\varrho}; \chi^s_t = \chi^s A_t^{1-\varrho}$.

First order conditions are:[4]

$$u_{ct} = (C_t - h\,C_{t-1})^{-\varrho} - \beta h\,(C_{t+1} - h\,C_t)^{-\varrho} \tag{3}$$

$$u_{ct}\,w_{ut} = \chi_t^u\,(L_{ut})^{\varphi} \tag{4}$$

$$u_{ct}\,w_{st} = \chi_t^s\,(L_{st})^{\varphi} \tag{5}$$

$$E_t \Lambda_{t,t+1} \pi_{t+1}^{-1} R_t^d = 1 \tag{6}$$

where u_{ct} highlights the marginal utility of consumption expressed in real terms, w_{ut}, w_{st} the real wage for each type of labour. The stochastic discount factor is:

$$\Lambda_{t,t+1} = \beta \frac{u_{ct+1}}{u_{ct}} \tag{7}$$

2.2 Bonds Structure

Long-term bonds are issued by the corporate sector (namely, wholesale producers and innovators) and the government, as a financing tool for their business. According to the structure presented in Sims & Wu (2021), bonds are perpetual contracts that guarantee decaying coupon payments.[5] Focusing on the innovator n issuing bonds as a general example, the flow representing the ultimate bond issuance is represented by CZ_{nt}, while the total amount of coupon liabilities is described by the following infinite sum of terms:

$$F_{nt-1}^z = CZ_{nt-1} + \kappa_Z\,CZ_{nt-2} + \kappa_Z^2\,CZ_{nt-3} + \ldots \tag{8}$$

[4]All analytical derivations are available in the technical appendix A.

[5]One bond issued at price Q_t at time t, guarantees the payment of 1 money unit in $t+1$, κ units in $t+2$, etc., with the coupon growing exponentially in time. In this framework, κ denotes the decay parameter of coupon payments.

This feature permits to control the new bond issuance as the difference in the stock between the last two periods, because an iteration of the previous expression one period forward gives:

$$CZ_{nt} = F^{z}_{nt} - \kappa_Z \, F^{z}_{nt-1} \tag{9}$$

Building on this definitions, identifying by F^{k}_{mt} the bonds issued by wholesale producer m and by B_{Gt} those belonging to the government, it is possible to express the value of bond portfolios by issuer, as follows:

$$Q_{Zt}F^{z}_{mt} = Q_{Zt}\, CZ_{nt} + \kappa_Z \, Q_{Zt}\, CZ_{nt-1} + \kappa_Z^2 \, Q_{Zt}\, CZ_{nt-2} + \dots \tag{10}$$

$$Q_{Kt}F^{k}_{mt} = Q_{Kt}\, CK_{mt} + \kappa_K \, Q_{Kt}\, CK_{mt-1} + \kappa_K^2 \, Q_{Kt}\, CK_{mt-2} + \dots \tag{11}$$

$$Q_{Bt}B_{Gt} = Q_{Bt}\, CB_{Gt} + \kappa_B \, Q_{Bt}\, CB_{Gt-1} + \kappa_B^2 \, Q_{Bt}\, CB_{Gt-2} + \dots \tag{12}$$

where Q_{Zt}, Q_{Kt}, Q_{Bt} represent bond prices and κ_Z, κ_K, κ_B decay coefficients of coupon payments.

2.3 Financial Intermediaries

Bankers follow the framework depicted in Sims & Wu (2021). Each intermediary i: finances its activity through net worth N_{it} and households' deposits D_{it}; holds private bonds issued by production firms F^{k}_{it} and innovators F^{z}_{it}, government bonds B_{it} and central bank's reserves RE_{it}. The mass of bankers is constant, such that every period a fraction $(1-\sigma)$ declares failure and returns his belongings to households, which then finance the opening of the same number of new banks through a start-up transfer X_t. Table 1 shows the balance sheet composition of a generic bank i, while net worth is given by:

$$Q_{Zt}F^{z}_{it} + Q_{Kt}F^{k}_{it} + Q_{Bt}B_{it} + RE_{it} = D_{it} + N_{it} \tag{13}$$

Assets	Liabilities
$Q_{Bt}B_{it}$	D_{it}
$Q_{Kt}F^k_{it}$	N_{it}
$Q_{Zt}F^z_{it}$	
RE_{it}	

Table 1: Bank i balance-sheet

Alive bankers accumulate net worth according to the following law of motion:

$$\begin{aligned} N_{it} = &\left(R^Z_t - R^d_{t-1}\right) Q_{Zt-1}F^z_{it-1} + \left(R^K_t - R^d_{t-1}\right) Q_{Kt-1}F^k_{it-1} + \\ &+ \left(R^B_t - R^d_{t-1}\right) Q_{Bt-1}B_{it-1} + \left(R^{re}_{t-1} - R^d_{t-1}\right) RE_{it-1} + R^d_{t-1}N_{it-1} \end{aligned} \tag{14}$$

Here, R^{re}_t is the gross interest rate applied by the monetary authority on reserves. It potentially differs from the deposit rate R^d_t, defined as a market clearing variable. Terms in round parentheses define excess returns on assets with respect to funding costs (i.e. the rate on deposits). Moreover, gross returns on assets can be defined as follows:

$$R^Z_t = \frac{1 + \kappa_Z Q_{Zt}}{Q_{Zt-1}} \tag{15}$$

$$R^K_t = \frac{1 + \kappa_K Q_{Kt}}{Q_{Kt-1}} \tag{16}$$

$$R^B_t = \frac{1 + \kappa_B Q_{Bt}}{Q_{Bt-1}} \tag{17}$$

According to Sims & Wu (2021), a representative banker maximizes the terminal franchise value:[6]

$$V_{it} = \max_{f^z_{it}, f^k_{it}, b_{it}, re_{it}} (1-\sigma)E_t \sum_{j=1}^{\infty} \sigma^{j-1}\Lambda_{t,t+j}n_{it+j} \tag{18}$$

subject to the usual incentive constraint, stating the condition for the management to

[6] Lower case definitions are real counterparts of nominal variables (i.e. $n_t = N_t/P_t$).

not divert funds:

$$V_{it} \geq \theta_t \left(Q_{Zt} f_{it}^z + \Delta_K Q_{Kt} f_{it}^k + \Delta_B Q_{Bt} b_{it} \right) \tag{19}$$

and a further reserves constraint, which defines the reserve requirement imposed by the central bank on the financial intermediary:

$$re_{it} \geq \varsigma_t d_{it} \tag{20}$$

In detail, constraint 19 tells the bank's value has to be higher than the potentially absconded funds, stating the condition to keep running the business. Here, θ_t quantifies the stochastic amount of bonds a banker can divert, while Δ_K, Δ_B scales this fraction by the effort required to divert each kind of asset.[7] Equation 20 shows the lowest level of reserves a bank must hold because of the binding regulatory activity. Here, this level is set proportional to deposits. Although the constraint is generally non-binding, it permits to generate a negative interest rate on reserves when binding.

Optimality conditions of the bank are:

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^Z - R_t^d \right) = \frac{\lambda_t}{1+\lambda_t} \theta_t \tag{21}$$

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^K - R_t^d \right) = \frac{\lambda_t}{1+\lambda_t} \theta_t \Delta_K \tag{22}$$

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^B - R_t^d \right) = \frac{\lambda_t}{1+\lambda_t} \theta_t \Delta_B \tag{23}$$

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_t^{re} - R_t^d \right) = -\frac{\omega_t}{1+\lambda_t} \tag{24}$$

where:

$$\Omega_t = 1 - \sigma + \sigma \theta_t \phi_t \tag{25}$$

[7] It is assumed easier to abscond private bonds than government ones, according to which $0 \leq \Delta_B < \Delta_K < 1$. Differently, θ_t behaves as a liquidity shock. Its increase means depositors have less power to recover funds after a bankruptcy, reducing their will to lend which translates into an interest rate spread increase, symptom of liquidity crises. In addition, reserves are fully recoverable.

$$\phi_t = \frac{1+\lambda_t}{\theta_t} E_t\left[\Lambda_{t,t+1}\Omega_{t+1}\pi_{t+1}^{-1}\right] R_t^d - \frac{\omega_t re_t}{n_t\theta_t} \tag{26}$$

The occurrence of binding constraints determines the existence of positive excess returns over the deposit rate. In particular, the presence of a binding incentive generates spreads on all kind of bonds, such that R&D spread will be higher than corporate and government ones. According to the calibration, this result converts in R&D projects featuring a higher return, consistent with the nature of the underlying activity. Contrarily, a binding reserve requirement might imply a reserve interest rate below the deposit one, however never being higher in principle. Finally, the event of non-binding constraints ensures the equality of all returns with the deposit rate, closing the spreads. Here, lagrangian multipliers on incentive and reserve constraints are, respectively, λ_t and ω_t. Equation 25 is an auxiliary definition and 26 represents the endogenous leverage ratio of a bank. In particular, in case of a binding incentive constraint, the leverage ratio is expressed under the following form:

$$\phi_t = \frac{Q_{Zt} f_{it}^z + \Delta_K Q_{Kt} f_{it}^k + \Delta_B Q_{Bt} b_{it}}{n_{it}} \tag{27}$$

This expression explains the suboptimality of the leverage degree of the bank with respect to the equilibrium level in 26, generating the existence of spreads. Furthermore, the franchise value can be exposed as:

$$V_{it} = \theta_t \phi_t n_{it} \tag{28}$$

and it is possible to rearrange the equilibrium leverage from 26 to show that in absence of binding constraints, irrelevance applies to the bank investment choice, as excess returns are missing.[8]:

$$\theta_t \phi_t = 1 + \lambda_t - \frac{\omega_t re_t}{n_t} \tag{29}$$

Thus, the franchise value is asymmetrically impacted from the constraints. The costly

[8]This result derives from observing the lagrangian multipliers converge to 0, such that $\theta_t \phi_t = 1$.

enforcement limit determines a positive value for λ_t, that transmits as a factor that increases $\theta_t\phi_t$ and creates the excess return for bonds. This means net worth has a higher potential within the bank disposal instead of being in the household availability, as this investment opportunity is not available for the latter. At odds, the reserve cap generates a loss in the franchise balance sheet which grows in the amount of held reserves, because it depresses the reserve interest rate below the deposit rate.

2.4 Corporate Sector

2.4.1 Capital Producers

Capital producers operating in a perfect competition regime invest in physical capital I_t^k, I_t^z, to supply intermediate firms and innovators, with the aim of selling the newly developed part at prices p_t^k, p_t^z. Their choice problem is:

$$\max_{I_t^k, I_t^z} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left\{ p_{t+j}^k I_{t+j}^{Nk} + p_{t+j}^z I_{t+j}^{Nz} - I_{t+j}^k - I_{t+j}^z \right\} \tag{30}$$

subject to the newly realized capital I_t^N, that according to Anzoategui et al. (2019), with g_t the steady-state growth rate of investments and $S(\cdot)$ a function defining their adjustment costs, is:

$$I_t^{Nk} = \left[1 - S\left(\frac{I_t^k}{I_{t-1}^k g_t}\right)\right] I_t^k \tag{31}$$

$$I_t^{Nz} = \left[1 - S\left(\frac{I_t^z}{I_{t-1}^z g_t}\right)\right] I_t^z \tag{32}$$

Then, first order conditions of the producers are:

$$p_t^k \left[1 - S\left(\frac{I_t^k}{I_{t-1}^k g_t}\right) - S'\left(\frac{I_t^k}{I_{t-1}^k g_t}\right) \frac{I_t^k}{I_{t-1}^k g_t}\right] + E_t \Lambda_{t,t+1} p_{t+1}^k S'\left(\frac{I_{t+1}^k}{I_t^k g_{t+1}}\right) \left(\frac{I_{t+1}^k}{I_t^k g_{t+1}}\right)^2 = 1 \tag{33}$$

$$p_t^z\left[1-S\left(\frac{I_t^z}{I_{t-1}^z g_t}\right)-S'\left(\frac{I_t^z}{I_{t-1}^z g_t}\right)\frac{I_t^z}{I_{t-1}^z g_t}\right]+E_t\Lambda_{t,t+1}p_{t+1}^z S'\left(\frac{I_{t+1}^z}{I_t^z g_{t+1}}\right)\left(\frac{I_{t+1}^z}{I_t^z g_{t+1}}\right)^2=1 \tag{34}$$

2.4.2 Wholesale Producers

A continuum of measure A_t of intermediate (wholesale) firms hires unskilled labour and new capital in order to produce output X_{mt}. Building on the horizontal growth paradigm (Romer, 1990), A_t represents the number of technological varieties available in the economy. Regarding the structure modelled in Anzoategui et al. (2019), key departure lies in the way producers finance themselves, relying on the issuance of long term bonds as described in Section 2.2.

The representative intermediate firm m produces by means of the following production function:

$$X_{mt}=\epsilon_t^A\left(u_t K_t\right)^{\alpha}\left(L_{ut}\right)^{1-\alpha} \tag{35}$$

with u_t the utilization rate of physical capital, K_t the stock of already owned capital, L_{ut} the quantity of unskilled labour and ϵ_t^A an exogenous TFP shock. In aggregate terms, intermediate output is:

$$X_t=\left[\int_0^{A_t} X_{mt}^{1/\vartheta}\,dm\right]^{\vartheta} \tag{36}$$

where ϑ is the markup desired by wholesale producers. Thus, each firm maximizes the actual value of the future stream of profits, deriving from selling output at price P_{mt}, subject to the law of motion for capital (which includes depreciation $\delta(u_t)$):

$$K_{t+1}=I_t^{Nk}+\left(1-\delta\left(u_t\right)\right)K_t \tag{37}$$

and a loan-in-advance constraint, which introduces the concept a firm is required to borrow through long-term bonds, in order to finance a fraction ψ^k of its investment:

$$\psi^k P_t^k I_t^{Nk} \leq Q_{Kt}\, CK_{mt} = Q_{Kt}\left(F_{mt}^k - \kappa_K F_{mt-1}^k\right) \tag{38}$$

In real terms, optimality conditions for labour, utilization of capital, stock of capital, bonds and new purchased capital are:

$$\mathcal{M} w_{ut} = (1-\alpha) p_{mt} \frac{X_{mt}}{L_{ut}} \tag{39}$$

$$\mathcal{M} p_t^k M_{1t} \delta'(u_t) K_t = \alpha p_{mt} \frac{X_{mt}}{u_t} \tag{40}$$

$$\mathcal{M}\left[p_t^k M_{1t} - E_t \Lambda_{t,t+1}\left(1-\delta\left(u_{t+1}\right)\right) p_{t+1}^k M_{1t+1}\right] = E_t \Lambda_{t,t+1}\left[\alpha p_{mt+1} \frac{X_{mt+1}}{K_{t+1}}\right] \tag{41}$$

$$\mathcal{M}\left[Q_{Kt} M_{2t} - E_t \Lambda_{t,t+1} \pi_{t+1}^{-1} \kappa_K Q_{Kt+1} M_{2t+1}\right] = E_t \Lambda_{t,t+1} \pi_{t+1}^{-1} \tag{42}$$

$$\frac{M_{1t}-1}{M_{2t}-1} = \psi^k \tag{43}$$

where all expressions include the desired markup $\mathcal{M}$, smaller than the optimal unconstrained markup ϑ to avoid the threat of entry by imitators. The last condition is obtained from rearranging the optimal choice for new incoming capital with the lagrangian multipliers attached to the constraints.[9] In detail, M_{1t}, M_{2t} can be interpreted as endogenous wedges on investments and financial conditions, whose intertemporal changes, namely variations in asset pricing decisions, determine the transmission of UMP onto the real economy.[10]

Following Anzoategui et al. (2019), production of the intermediate output can be aggregated, expressing final output in the following terms:[11]

[9]Characterizing with v_{1t} the multiplier on the capital constraint, and v_{2t} the one on the loan-in-advance constraint, we define: $M_{1t} = 1 + \psi^k v_{2t}$, and $M_{2t} = 1 + v_{2t}$. In addition, we obtain: $v_{1t} = p_t^k M_{1t}$.

[10]In case of non-binding constraints, both wedges reduce to 1 ($M_{1t} = M_{2t} = 1$) and asset pricing conditions converge to standard ones.

[11]See the technical appendix for further details on the aggregation.

$$Y_t = A_t^{\vartheta-1}\epsilon_t^A \left(u_t K_t\right)^{\alpha} \left(L_{ut}\right)^{1-\alpha} \tag{44}$$

where total factor productivity is augmented by an endogenous component: $A_t^{\vartheta-1}$.

2.4.3 Final Good Producers

Intermediate output good is purchased by f monopolistically competitive final good producers, which transform the intermediate-goods composite into final output according to a linear technology function:

$$Y_{ft} = X_{ft} \tag{45}$$

such that the final good realized by firm f is equal to the intermediate input adopted. Aggregate final output is thus the mass-one CES aggregator of the existent final goods:

$$Y_t = \left[\int_0^1 Y_{ft}^{1/\mu}\, df\right]^{\mu} \tag{46}$$

where μ is the imposed markup. Given retailers employ intermediate inputs as the only productive factor, it is possible to introduce their real marginal costs, set upon the aggregate of the relative intermediate prices:

$$mc_t = \frac{p_{mt}}{A_t^{\vartheta-1}} \tag{47}$$

The optimal price choice is influenced by a Calvo nominal rigidity, where only a fraction $(1-\omega_p)$ of firms can freely adjust its price to the optimal level, while the remaining index to the lagged inflation rate. Thus, the firm pricing problem is:

$$\max_{P_{ft}^*} E_t \sum_{j=0}^{\infty} \omega_p^j \Lambda_{t,t+j} \left(\frac{P_{ft}^*}{P_{ft+j}}\Gamma_{t,t+j} - mc_{t+j}\right) Y_{ft+j} \tag{48}$$

where: $\Gamma_{t,t+j} = \prod_{\tau=1}^{j} \left(\pi_{t+\tau-1}\right)^{\iota^{\pi}} \overline{\pi}^{1-\iota^{\pi}}$, represents the indexation rule, ι^{π} offers a measure of the price indexation and $\overline{\pi}$ is the steady-state inflation rate. Firms unable to reset optimally, adjust prices as:

$$P_{ft} = P_{ft-1}\pi_{t-1}^{\iota^{\pi}}\overline{\pi}^{1-\iota^{\pi}} \tag{49}$$

Moving to real terms, the first order condition for $p_t^*(= P_{ft}^*/P_{t-1})$ is:

$$E_t\sum_{j=0}^{\infty}\omega_p{}^j\Lambda_{t,t+j}\left(\frac{p_t^*}{\pi_{t+j}}\Gamma_{t,t+j}\right)^{\frac{\mu}{1-\mu}}\left(\frac{p_t^*}{\pi_{t+j}}\Gamma_{t,t+j}-\mu\, mc_{t+j}\right)Y_{ft+j} = 0 \tag{50}$$

In aggregate terms, inflation dynamics are:

$$\pi_t = \left\{\left(1-\omega_p\right)\left(p_t^*\right)^{\frac{1}{1-\mu}} + \omega_p\left(\pi_{t-1}^{\iota^{\pi}}\overline{\pi}^{1-\iota^{\pi}}\right)^{\frac{1}{1-\mu}}\right\}^{1-\mu} \tag{51}$$

2.5 Labor Market

Employment agencies intermediate the households supply of skilled and unskilled labour, which is hired by firms under the form of a labour composite. Denoting with l_t the composite for each typology of labour ($l_t = \{L_{st}, L_{ut}\}$), this represents CES-aggregated labour provided by the h households:

$$l_t = \left[\int_0^1 l_{ht}^{1/\mu_w}\,dh\right]^{\mu_w} \tag{52}$$

A competitive employment agency maximizes profits, under a Calvo wage rigidity. A group made of $(1-\omega_w)$ households re-optimizes wages each period and sets the optimal reset wage W_{lt}^* through the following maximization problem:

$$\max_{W_{lt}^*} E_t\sum_{j=0}^{\infty}\omega_w^j\left[-\chi_l\frac{\left(l_{ht+j}\right)^{1+\varphi}}{1+\varphi} + u_{ct+j}\frac{W_{lt}^*\Gamma_{wt,t+j}}{P_{t+j}}l_{ht+j}\right] \tag{53}$$

where the indexation rule is depicted as: $\Gamma_{wt,t+j} = \prod_{\tau=1}^{j}\left(\pi_{t+\tau-1}\right)^{\iota^w}\overline{\pi}^{1-\iota^w}g_t$. The remaining fraction obeys the indexation rule which follows:

$$W_{lt} = W_{lt-1}\pi_{t-1}^{\iota^w}\overline{\pi}^{1-\iota^w}g_t \tag{54}$$

Switching to real terms, the optimality condition emerging from the problem is:

$$E_t \sum_{j=0}^{\infty} \omega_w^j \Lambda_{t,j} \left[\frac{w_{lt}^* \Gamma_{wt,t+j}}{\prod_{\tau=1}^{j} \pi_{t+\tau}} - \mu_w \chi_{lt} \left(\frac{w_{lt}^* \Gamma_{wt,t+j}}{w_{lt+j} \prod_{\tau=1}^{j} \pi_{t+\tau}} \right)^{\frac{\varphi \mu_w}{1-\mu_w}} \frac{l_{t+j}^{\varphi}}{u_{ct+j}} \right] * \\ * \left(\frac{w_{lt}^* \Gamma_{wt,t+j}}{w_{lt+j} \prod_{\tau=1}^{j} \pi_{t+\tau}} \right)^{\frac{\mu_w}{1-\mu_w}} l_{t+j} = 0 \tag{55}$$

while the rule describing each aggregate wage composite is:

$$w_{lt} = \left\{ (1-\omega_w) \left(w_{lt}^* \right)^{\frac{1}{1-\mu_w}} + \omega_w \left(g_t \pi_{t-1}^{\iota_w} \overline{\pi}^{1-\iota_w} \frac{w_{lt-1}}{\pi_t} \right)^{\frac{1}{1-\mu_w}} \right\}^{1-\mu_w} \tag{56}$$

2.6 Innovators

Following Queralto (2020), n competitive innovators develop new technological varieties through the use of capital good I_t^{Nz} and skilled labour L_{st}. Each innovator releases its production as rights-to-the-use of the new varieties. Similar to wholesale producers, innovators face an inefficient availability of funds in order to cover R&D efforts. This generates a financial friction under the form of a borrowing constraint, as innovators borrow resources from banks. Again, loans are covered trough the issuance of long-term bonds F_{nt}^z. Thus, innovators maximize real incoming dividends:

$$\max_{F_{nt}^z, I_t^{Nz}, L_{st}} Q_t Z_t - p_t^z I_t^{Nz} - w_{st} L_{st} + Q_{Zt} \left(\frac{F_{nt}^z}{P_t} - \kappa_z \frac{F_{nt-1}^z}{P_{t-1}} \pi_t^{-1} \right) - \frac{F_{nt-1}^z}{P_{t-1}} \pi_t^{-1} \tag{57}$$

under the constraints represented by the production function for new technologies:[12]

$$Z_t = \left(I_t^{Nz} \right)^{\eta} \left(A_t L_{st} \right)^{1-\eta} \tag{58}$$

and the loan-in-advance constraint, where a fraction of the final good employed as productive input is bank-financed:

[12] It embodies the aggregate technological level A_t as a positive externality increasing labour efficiency.

$$\psi^z p_t^z I_t^{Nz} \leq Q_{Zt} \left(\frac{F_{nt}^z}{P_t} - \kappa_Z \frac{F_{nt-1}^z}{P_{t-1}} \pi_t^{-1} \right) \tag{59}$$

Optimality conditions are:

$$w_{st} = M_{3t} p_t^z \left(\frac{1-\eta}{\eta} \right) \frac{I_t^{Nz}}{L_{st}} \tag{60}$$

$$Q_{Zt} M_{4t} = E_t \Lambda_{t,t+1} \pi_{t+1}^{-1} \left[1 + Q_{Zt+1} \kappa_Z M_{4t+1} \right] \tag{61}$$

$$\frac{M_{3t} - 1}{M_{4t} - 1} = \psi^z \tag{62}$$

where the last equation emerges from a rearrangement of the first-order condition for final ouptut and the lagrangian multipliers.[13] Again, M_{3t}, M_{4t} represent endogenous wedges on production investment and financial conditions, key for the display of unconventional policies.

Endogenous technology evolves as the non-depreciated part of existing and new technological varieties:

$$A_{t+1} = \left(1 - \delta^A\right) \left(A_t + Z_t\right) \tag{63}$$

and the TFP growth rate is:

$$g_{t+1} = \frac{A_{t+1}}{A_t} = \left(1 - \delta^A\right)\left(1 + \frac{Z_t}{A_t}\right) \tag{64}$$

2.7 Government

The government collects lump-sum taxes from the fiscal imposition over households T_t, a transfer from the monetary authority T_{cbt} and revenues accruing from the bonds

[13] Denoting v_{3t}, v_{4t} the multipliers on innovation production and borrowing constraint, we obtain: $M_{3t} = 1 + \psi^z v_{4t}$, and $M_{4t} = 1 + v_{4t}$. In addition: $v_{3t} = M_{3t} p_t^z / \left[\eta \left(I_t^{Nz} \right)^{\eta - 1} \left(A_t L_{st} \right)^{1-\eta} \right]$.

issuance B_{Gt}. Collected revenues finance a stochastic amount of public spending G_t.[14]

The government real budget constraint is given by:

$$G_t + \frac{\overline{b}_G}{\pi_t} = T_t + T_{cbt} + Q_{Bt}\overline{b}_G \left(1 - \kappa_B \pi_t^{-1}\right) \tag{65}$$

2.8 Monetary Policy

First, the central bank sets conventional monetary policy through a Taylor Rule, that controls the short term interest rate R_t^{TR} in the following way:

$$\ln R_t^{TR} = \rho_r \ln R_{t-1}^{TR} + (1-\rho_r)\left[\ln \overline{R}^{TR} + \phi_\pi \left(\ln \pi_t - \ln \overline{\pi}\right) + \phi_y \left(\ln mc_t - \ln \overline{mc}\right)\right] + s_r \varepsilon_{rt} \tag{66}$$

with $\overline{R}^{TR}, \overline{\pi}$ the steady-state policy rate and inflation target, ϕ_π, ϕ_y the weights attached on controlling each target, ρ_r the smoothing parameter of the monetary action. According to Anzoategui et al. (2019), the rule targets deviations of marginal costs from the balanced growth path level, as a proxy of the output gap.

In normal times, i.e. periods in which the monetary conduct is not limited by the ZLB, the central bank fixes the reserve interest rate equal to the above policy rate. Moreover, the reserve requirement does not bind and the next equilibrium condition emerges:

$$R_t^d = R_t^{re} = R_t^{TR} \tag{67}$$

Differently, ZLB periods, which justify the implementation of the subsequent types of unconventional monetary policies, are connoted by a net policy interest rate forbidden to go below zero. Thus, assuming again the case of a non-binding requirement on reserves, the ZLB constraint implies the equality between deposit and reserve rates, where the latter has now to be positive:

[14] It is assumed the quantity of real government bonds as fixed, $b_{Gt} = \overline{b}_G$, with nominal bonds growing at the price level, $B_{Gt} = P_t\overline{b}_G$. The amount of taxes automatically adjusts as to clear the fiscal budget constraint each period.

$$R_t^d = R_t^{re} = \max\{1,\, R_t^{TR}\} \tag{68}$$

2.9 Unconventional Monetary Policy

Three different kinds of unconventional monetary policy are analyzed within this model economy: wide purchases of private and public bonds carried out by the central bank as an emergency lending tool, i.e. QE; communications from the central bank highlighting the planned future path of policy rate movements, i.e. FG; the option of setting policy rates in negative territory, i.e. NIRP.

2.9.1 Quantitative Easing

QE bonds purchases by the central bank are covered through the creation of reserves, which are held by banks and remunerated. Therefore, it is possible to describe the central bank's balance sheet equilibrium:

$$Q_{Zt} f_{cbt}^z + Q_{Kt} f_{cbt}^k + Q_{Bt} b_{cbt} = re_t \tag{69}$$

where held bonds equal the issuance of interest-bearing reserves and any further revenue constitutes a transfer directed to the government.

To begin, I analyze the case for exogenous QE measures, where the central bank's bonds purchase is governed by the conditions below:

$$f_{cbt}^z = (1-\rho_z)\, f_{cb}^z + \rho_z f_{cbt-1}^z + s_z \varepsilon_{zt} \tag{70}$$

$$f_{cbt}^k = (1-\rho_k)\, f_{cb}^k + \rho_k f_{cbt-1}^k + s_k \varepsilon_{kt} \tag{71}$$

$$b_{cbt} = (1-\rho_b)\, b_{cb} + \rho_b b_{cbt-1} + s_b \varepsilon_{bt} \tag{72}$$

Here, $f_{cb}^z, f_{cb}^k, b_{cb}$ are steady-state bond holding levels and ρ, s, denote persistence of the AR(1) processes and standard deviation of the shocks.

2.9.2 Forward Guidance

The central bank purpose of anchoring agents' expectations is executed through communications. As sketched in Sims & Wu (2021), forward guidance activity is here modelled under the form of a shock $(s_r \varepsilon_{rt})$ that impacts the policy rate R_t^{TR}, set by the conventional monetary rule in equation 66. As an unconventional tool, it is implemented in crisis periods, i.e. when the ZLB represented in equation 68 binds. Provided the length of the ZLB is given, this structure implies that a shock capable to reduce the desired policy rate also determines a fall in the reserve and deposit rates, at the end of the ZLB. Thus, today's shocks influence current long-term rates as long as agents believe the communications, because of the rational expectations framework.[15]

2.9.3 Negative Interest Rate Policy

The experiment of bringing policy rates in negative territory is here represented as follows. The monetary authority decides the level of the reserve interest rate R_t^{re} equals the Taylor Rule rate R_t^{TR}, with no downward limit.[16] At the same time, a constraint on the deposit rate is imposed, such that equation 68 becomes:

$$R_t^d = \max\{1,\, R_t^{re}\} \quad \text{with} \quad R_t^{re} = R_t^{TR} \tag{73}$$

which means the ZLB only binds on the deposit rate, leaving the reserve rate free to adjust. NIRP enters as a shock that brings the desired policy rate down, inducing the same adjustment on the reserve rate as they are tied, while the deposit rate is stuck at 0 by construction and the reserve requirement becomes binding. This feature stems from the need to regulate banks' ability to hold reserves, as they would collect an infinite amount at the equality condition: $R_t^{re} = R_t^d$, and none if inequality between the two holds $(R_t^{re} < R_t^d)$. Therefore, the central bank disciplines the intermediaries' behaviour against their propensity, by imposing to observe a reserve requirement.

[15] I refer the reader to Sims & Wu (2021) for the differences with respect to common ways of modelling forward guidance and the ability of this structure to avoid the *"forward guidance puzzle"*.

[16] I leave the analysis of the effective lower bound to future work.

2.10 Market Clearing & Aggregation

The outlined model includes three autoregressive of order one shock processes, mimicking exogenous innovations to TFP, liquidity and fiscal spending:

$$\ln \epsilon_t^A = \rho_A \ln \epsilon_{t-1}^A + s_A \varepsilon_{At} \tag{74}$$

$$\ln \theta_t = (1-\rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + s_\theta \varepsilon_{\theta t} \tag{75}$$

$$\ln G_t = (1-\rho_G) \ln G + \rho_G \ln G_{t-1} + s_G \varepsilon_{Gt} \tag{76}$$

where G, θ are deterministic steady-state values for government spending and liquidity. The labour market is composed of skilled and unskilled workers:

$$L_t = L_{st} + L_{ut} \tag{77}$$

Market clearing conditions to all bond markets apply:

$$f_{nt}^z = f_t^z + f_{cbt}^z \tag{78}$$

$$f_{mt}^k = f_t^k + f_{cbt}^k \tag{79}$$

$$\overline{b}_G = b_t + b_{cbt} \tag{80}$$

where f_t^z, f_t^k, b_t are obtained summing each single held amount across the i intermediaries. The aggregation of financial intermediaries' individual balance sheets gives the following relation:

$$Q_{Zt} f_t^z + Q_{Kt} f_t^k + Q_{Bt} b_t + re_t = d_t + n_t \tag{81}$$

while the net worth law of motion at aggregate level is:

$$\begin{aligned} n_t = \sigma \pi_t^{-1} [\left(R_t^Z - R_{t-1}^d\right) Q_{Zt-1} f_{t-1}^z + \left(R_t^K - R_{t-1}^d\right) Q_{Kt-1} f_{t-1}^k + \\ + \left(R_t^B - R_{t-1}^d\right) Q_{Bt-1} b_{t-1} + \left(R_{t-1}^{re} - R_{t-1}^d\right) re_{t-1} + R_{t-1}^d n_{t-1}] + H_t \end{aligned} \tag{82}$$

Similarly, it is possible to express the relationship between aggregate net worth and the sector's leverage ratio:

$$Q_{Zt} f_t^z + \Delta_K Q_{Kt} f_t^k + \Delta_B Q_{Bt} b_t \leq \phi_t n_t \tag{83}$$

while the economy features the following aggregate resource constraint:

$$Y_t = C_t + I_t^k + I_t^z + G_t \tag{84}$$

2.11 Model Solution & Calibration

First, the model is made stationary detrending each variable with respect to its deterministic balanced growth path. According to Queralto (2020), variables are stationarized by the level of technology A_t. The full stationary model is presented in the technical appendix A. Then, I solve the model computing a log-linear first order approximation around the steady state of the stationary system. The occurrence of the ZLB is studied through a piecewise linear approximation following Guerrieri & Iacoviello (2015).

Model-relevant calibrated parameters are described in table 2.[17] Standard values apply for: the discount factor β, habit formation h, Frisch elasticity φ, banks' survival probability σ, the capital share α, adjustment costs on capital investments κ_I, fractions resetting prices and wages and indexation levels ($\omega_p, \omega_w, \iota^\pi, \iota^w$), the steady-state tangible and intangible capital depreciation levels (δ, δ^A), inflation target and Taylor rule reaction coefficients ($\overline{\pi}, \phi^\pi, \phi^y$). I fix the overall labour supply to 1/3 and calibrate the share of skilled workers as the 12.8% of the aggregate, according to the *"NSF Business R&D and Innovation Survey"*. Skilled and unskilled labour disutilities match these targets. TFP quarterly growth rate is compatible with an output net annual growth rate of 1.8%, as estimated by Anzoategui et al. (2019). Target for the spread on R&D capital is the difference between *"BofA BB US High Yield Index"* and the Federal Funds Rate, while the one on physical capital derives from the *"BofA AAA Corporate Bond Yield"* minus the FFR. Spread on government bonds is the excess return of the 10-year Treasury yield over the FFR. From these targets, I obtain the BGP level for θ, higher than common values

[17] The complete list of calibrated parameters is reported in appendix, table 3.

Parameter	Definition	Value / Target *	Source
Households			
χ^u	Unskilled Labour Disutility	* L = 0.33	Literature
χ^s	Skilled Labour Disutility	* $\overline{L_s}/L = 12.8\%$	*NSF*
Bankers			
σ	Survival Rate	0.950	Sims & Wu (2021)
θ	Recoverability parameter	* Leverage = 4	"
κ_B	Government Bond Duration	* $(1-\kappa_B)^{-1} = 40$	"
κ_K	Private Bond Duration	* $(1-\kappa_K)^{-1} = 40$	"
κ_Z	Innovation Bond Duration	* $(1-\kappa_Z)^{-1} = 40$	"
Δ_B	Government Bond Recoverability	* $(RK - Rd) = 0.0075$	Bonciani et al. (2023)
Δ_K	Private Bond Recoverability	* $(RZ - Rd) = 0.0115$	"
Non-financial firms			
ψ^k	Fraction of Investment from Debt	0.800	Sims & Wu (2021)
ϑ	Desired Markup on Interm. Good X	1.670	Queralto (2020)
μ	Markup on Final Good Y	1.100	Anzoategui et al. (2019)
$\mathcal{M}$	Effective Markup on Interm. Good X	1.180	"
μ_w	Markup on Wages	0.150	"
Technology Sectors			
ψ^z	Fraction of Investment from Debt	0.700	
δ^A	Technology Depreciation (SS)	0.030	Bonciani et al. (2023)
η	Capital Share in Innovation	0.190	Queralto (2020)
Central Bank			
b_{cb}	CB Treasury Holdings (SS)	0.060	Sims & Wu (2021)
f^k_{cb}	CB Private Bond Holdings (SS)	0	"
f^z_{cb}	CB Innovation Bond Holdings (SS)	0	"
b_G	Government Debt (SS)	0.410	"
Shock Processes			
ρ_θ	Liquidity Shock Persistence	0.980	Sims & Wu (2021)
ρ_b	Treasury Holdings Persistence	0.980	"
ρ_k	Private Bond Persistence	0.800	"
ρ_z	Innovation Bond Persistence	0.800	
s_r	Monetary Shock SD	0.0025	Sims & Wu (2021)
s_θ	Liquidity Shock SD	0.040	"
s_b	Treasury Holding SD	0.000	"
s_k	Private Bond SD	0.0025	"
s_z	Innovation Bond SD	0.0025	

Table 2: Calibrated Parameters

because of the easiness to abscond capital devoted to intangible output projects, and the recoverability parameters Δ_B and Δ_K, given by the ratio of the respective spreads over the R&D one. Values of κ_B, κ_K, κ_Z define the bonds' duration, where a target of 40 quarters is set to define long-term bonds. Production firms' markups follow the values set by Anzoategui et al. (2019) and Queralto (2020). Sims & Wu (2021) is the source to calibrate parameters defining the capital producers' block and the fraction of debt-financed investment ψ^k. This value represents the upper bound for its innovation counterpart, ψ^z, as innovation relies less on external-financing. Government spending matches the share of 20% of output, while debt matches a ratio debt-to-GDP equal to 41%. BGP values of central bank private bond holdings $\left(f_{cb}^{k}, f_{cb}^{z}\right)$ amount to zero, as the *Federal Reserve* started holding corporate debt only during the GFC. Contrarily, holdings of government bonds b_{cb} match the asset share with respect to annual GDP, observed before the GFC (6%). The intangible capital share in innovation η is the value estimated by Queralto (2020). Shocks are calibrated in line with Sims & Wu (2021). I assume the same level of persistence and standard deviation across exogenous private bonds purchases.

3 The Long-Run Effects of UMP Measures

I here study the long-run effects induced by unconventional monetary policy measures. I begin by analyzing the aftermath of a conventional monetary policy innovation, also thought as a baseline reference for evaluating desirability and magnitude of the alternative stabilization tools. Then, results are compared with unconventional measures. Building on Sims & Wu (2021), QE, FG and NIRP are activated assuming the economy is brought within a ZLB period because of the subsequent hitting of several liquidity shocks. In addition, the environment is supposed to be trapped under ZLB for 10 quarters, similar to the expected duration estimated in Wu & Xia (2016). Unconventional shocks are calibrated as to broadly mimic the output increase generated at impact by a conventional shock. Figure 4 reports the impulse response functions.

3.1 Conventional Monetary Policy

The conventional monetary policy shock is introduced as an exogenous variation to the Taylor Rule depicted in equation 66, which hits after 6 quarters and amounts to -1% on an annual basis. This shock translates into a reduction of the policy rate R^{TR}, equal to -0.75% on impact. We observe common dynamics for aggregate variables, where output, consumption expenditure and inflation increase after the shock (blue solid line, figure 4). Being outside of the ZLB, reserve (i.e. policy) and deposit rates share the same pattern.

Focusing on the credit-channel of innovation, as the nominal yield on the long-term

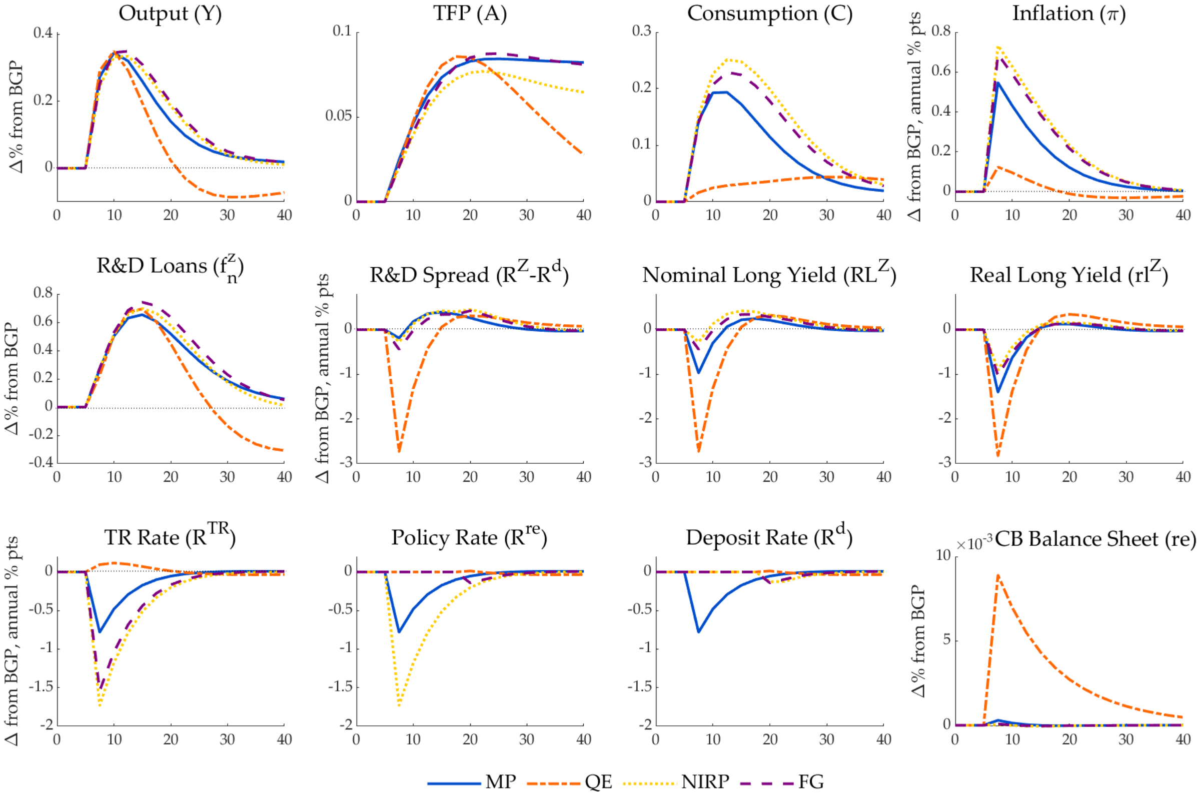

Figure 4: Impulse Responses to Monetary Policy Shocks

Impulse responses for: conventional monetary policy (blue solid), quantitative easing (orange dashed), forward guidance (purple dashed), negative interest rate policy (yellow dotted).

bond issued by the innovators collapses more than the deposit rate, the spread on loans devoted to finance intangible output projects initially shrinks. This easing of credit conditions transmits on the amount of released loans, which shows an evident increase over time. The effect is quite delayed, as offered by an intertemporal comparison between the dynamics of loans and the spread. The latter adjusts faster, in both its expansionary and contractionary phases. This credit boom, fueling the innovation effort, determines a steady increase of total factor productivity, enduring for more than 10 years after the policy implementation. Similarly, output is influenced for longer than what explained by common model specifications, which do not consider the credit-financing of R&D. However, the persistence of the effect is milder with respect to TFP, as the aggregate demand expansion driven by the increase in private consumption almost vanishes around the border between the short- and long-run.

3.2 Quantitative Easing

The central bank can execute QE as a purchase of private and government bonds. In the simulations illustrated (orange dashed line, figure 4), I study the specific case of a QE targeting bonds issued by the innovators.[18] The acquisition happens on the secondary market, where bonds already held by commercial banks are exchanged with interest bearing reserves provided by the central bank. The purchased amount of bonds consists in a central bank balance sheet expansion of almost 1 percentage point. From a comparative analysis, QE seems to spur business cycle aggregates in the same way of the interest rate steering activity implemented in normal times. At odds, output grows but its behaviour is more short-lived and the effects vanish soon. The boost in private consumption is milder and slower than that provoked by the alternatives because QE does not exert an impact on the deposit rate, which is the driver of the households' consumption choice as described by the Euler equation (6). For the same reason, QE generates a spike in inflation that is below that induced by other policy options considered. The transmission channel on the supply side shows similarities with the conventional policy one. The expansionary potential displays through the enlargement of the central bank balance sheet, which translates in a wider liquidity availability at the banks' disposal and a consequent reduction of the credit spread. This time, the spread falls deeper, as the deposit rate is unaltered during the ZLB period. The variation is again huge and fast. The one-off nature of the policy, which implies an immediate readjustment in the level of issued reserves, is suddenly mirrored in the financial intermediaries behaviour. Banks handle this scenario by reducing the credit provision, reflecting an upcoming tightening in their resource constraint. Therefore, the push on realizing new technologies is less prolonged. Contrarily to what observed for the other events, the TFP spike which follows is not permanent, even though highly persistent, likewise more smoothed is its spillover on GDP.

According to the literature building on Gertler & Karadi (2011), the incentive constraint defined in equation 19 rules the presence of real effects deriving from this policy. Indeed, banks under a binding constraint ease their leverage position as the central bank issues

[18] The choice aims to highlight the policy effect on innovation. However, a more comprehensive analysis would focus on a mixed portfolio composition among all bonds typologies.

reserves to buy bonds. In this scenario, the acquisition of bonds by the central bank does not displace intermediaries' bond purchases, while it alters pricing conditions as the price of bonds goes up, thus helping entrepreneurs to alleviate the limiting loan-in-advance constraint. This mechanism generates more investment, ending into a higher volume of aggregate demand and increasing TFP. Alternatively, QE would not produce an outcome in the cases of non-binding costly enforcement constraint, or when firms had enough internal resources to avoid the debt-issuance to finance investments (i.e. non-binding loan-in-advance constraint). However, assuming the realistic hypothesis of an environment in which both constraints bind, QE purchases determine real consequences. Effects are bigger the more the purchase targets private bonds, and they impact the economy on longer frequencies the more they sustain the innovation sector.[19]

3.3 Forward Guidance

Forward guidance implementation coincides with an exogenous shock to the Taylor Rule, in the magnitude of a 1.8% reduction that is converted in a 1.5% point fall of the desired policy rate, on impact (purple dashed line, figure 4). Consistent with the nature of the policy, FG communications do not directly influence the deposit rate in the meanwhile of a ZLB period. The deposit rate is influenced by the measure only after the ZLB ends. This delay, determined by the binding constraint, implies the need for a bigger current intervention under the form of FG, in order to obtain an impact response of output comparable to that observed after a conventional policy implementation. Thus, the required strength of forward guidance is proportional to the expected length of the ZLB. Moreover, even if delayed, the impact of the policy on the deposit rate justifies a higher inflationary potential, contrarily to QE.

The witnessed positive effects attributed to the policy suggest a strong support in favour of a central bank forward guiding economic agents. Simulations tell the goodness in terms of current and future outcomes, where the policy obtains the same stabilization results of conventional key policy rate reductions. This happens although the ZLB

[19] This result descends from being: $0 \leq \Delta_B < \Delta_K < 1$.

constraining this conventional instrument, thus theoretically preventing the monetary authority from exerting its full power. The transmission channel is now driven by the promise on future actions, but develops along the same path already inspected. Productivity increases as seen in the case of a classic monetary shock, being the consequence of a bankers' higher propensity to lend, visible in the enlargement of credit flows provided to R&D activities. Output shares the same behaviour and rationale, even though a bigger expansion follows from the consumption boom, higher than previously observed. In any case, the strength of FG communications (and generated results) have to be weighted by the credibility level of the central bank, which is here assumed to be perfectly reliable.[20]

3.4 Negative Interest Rate Policy

The negative interest rate policy here studied consists in a -2% shock to the annualized desired rate, that again amounts to an effective 1.8% reduction of desired and policy rates, while the net deposit rate is prevented to go below 0. Again, IRFs display a true aggregate expansion, consistent with the other policy options from both the qualitative and quantitative side (yellow dotted line, figure 4). In particular, conclusions from a long-run perspective are unaltered.

Nevertheless, the outcome of a NIRP implementation conveys from different channels. Initially, as this kind of policy requires a current reduction in the interest rate on reserves, it carries out a concrete action that represents a signal for a subsequent decrease of the deposit rate, at the conclusion of a ZLB spell. Thus, NIRP partly acts as forward guidance communications, reducing the credibility issue as NIRP relies on an observable measure.[21] However, the goodness of this mechanism is contrasted by a further channel, because of the relation between deposit and reserve interest rates. In a scenario of the former being bigger than the latter, it is visible from equation 14 how this inequality would translate into a cost for banks above the option of holding reserves. Therefore, this policy also determines a shrinkage in banks' net worth, worsening the

[20] I abstract from extending the analysis to different credibility levels of the central bank, as performed in Sims & Wu (2021), as this issue does not alter the conclusions on the long-run effects of FG.

[21] The similarity between NIRP and FG is true until the smoothing parameter driving the dynamics of the interest rate is high with respect to the length of the ZLB period.

incentive constraint. A preferred policy mix between FG and NIRP would thus need to consider the balance between the two channels, controlling for the credibility degree of the monetary authority and the amount of reserves held by private banks. Accordingly, under the hypothesis of implemented measures of a comparable size, NIRP would be less expansionary than FG. Bringing interest rates in negative territory induces the profit loss and tightening of the borrowing constraint. To compensate for the activation of this adverse channel, the NIRP shock applied is slightly bigger than FG one. However, impulse responses show that big differences between the two policies are absent, also in terms of the intervention magnitude, because in this model economy the BGP level of the central bank's balance sheet is of a negligible size. This means the amount of reserves within commercial banks' balance sheets is limited, while in a different scenario NIRP would generate a heavier toll on banks.

3.5 Rationale

In order to interpret and understand differences among the effects generated by competing policy options, it is useful to inspect their influence on long-term interest rates. I again consider the case of the innovator-issued bond, and define its gross yield RL_t^Z according to the next relation:[22]

$$RL_t^Z = \frac{1}{Q_{Zt}} + \kappa_Z \tag{85}$$

Thus, the total spread can be written as the distance between the net long-term yield and the respective rate on deposits:

$$ln\, RL_t^Z - ln\, R_t^d \tag{86}$$

where the credit spread on intangible capital is instead the gap between yields on

[22]By the definition of: $Q_{Zt} = \frac{1}{RL_t^Z} + \frac{\kappa_Z}{\left(RL_t^Z\right)^2} + \frac{\kappa_Z^2}{\left(RL_t^Z\right)^3} + ...$, it is possible to rearrange for RL_t^Z and obtain equation 85. The rationale equally applies to firms and government bonds.

private and public bonds:

$$ln\, RL_t^Z - ln\, RL_t^B \tag{87}$$

By switching to real terms, the long term return is:

$$r_t^L = \underbrace{ln\, RL_t^Z - ln\, R_t^d}_{\text{Overall Spread}} + \underbrace{ln\, R_t^d - E_t\, ln\, \pi_{t+1}}_{\text{Real Deposit Rate}} = ln\, RL_t^Z - E_t\, ln\, \pi_{t+1} \tag{88}$$

which is composed by the overall spread, inclusive of the term and credit components, and the deposit rate.

The long term private yield is a meaningful variable in that represents the paramount quantity to discern monetary policy transmission mechanisms in this context. In particular, output shares the same impact and dynamics after each of the implemented policy, as the real long yield is affected in a similar manner by the shocks. This is reflected in the impact of the interventions on the total spread in equation 86. Conventional monetary policy imposes a variation in short-term rates, which transmit onto longer ones. The transmission is such that long rates react in the same direction, shrinking the total spread. Contrarily, unconventional measures directly influence long yields but have no effect on the deposit rate, again closing the overall spread.

4 Conclusions

In this paper, we analyzed the transmission of unconventional monetary policy measures, where policy interventions are tied to credit-financed endogenous growth. Similarly to what observed in Beqiraj et al. (2025), the interaction among monetary policy and financial conditions steer long-term scenarios. Here, theoretical channels below this connection are highlighted for alternative policies. UMP is thus equally capable of determining aggregate macroeconomic effects on long horizons.

Core of the analysis is the realization of a dynamic general equilibrium model where R&D investments are financed by the financial intermediaries' credit provision, establishing a direct connection between financial conditions and endogenous growth mechanisms. The model is structured as to allow the joint study of different unconventional monetary policies, consisting in QE purchases, FG communications and NIRP. Besides the peculiarities associated to distinct policies, all expansionary interventions transmit through the credit channel. UMP reduce the credit spread, easing banks' financial conditions and the release of credit flows towards the private sector. When loans are directed to innovators, they affect the growth engine of an economy, determining enduring macroeconomic effects. In particular, under a calibration that induces the same output response across policies, FG and NIRP are the best options to permanently increase the level of productivity. Moreover, they can influence TFP and output as done by conventional measures, dampening the ZLB constraint in terms of aggregate stabilization. Finally, QE is able to generate a highly persistent push on TFP, even though quantitatively reduced with respect to other options, while its impact on output results to be more short-lived.

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A Technical Appendix

A.1 Model Derivation

Households

$$\max_{C_t,D_t,L_{ut},L_{st}} E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{\left(C_{t+j} - h\,C_{t+j-1}\right)^{1-\varrho}}{1-\varrho} - \chi_t^u \frac{\left(L_{ut+j}\right)^{1+\varphi}}{1+\varphi} - \chi_t^s \frac{\left(L_{st+j}\right)^{1+\varphi}}{1+\varphi} \right\}$$

s.t:

$$P_t C_t + D_t - D_{t-1} \leq W_{ut} L_{ut} + W_{st} L_{st} + DIV_t - P_t H_t - P_t T_t + \left(R_{t-1}^d - 1\right) D_{t-1}$$

$$\chi_t^u = \chi^u A_t^{1-\varrho}$$
$$\chi_t^s = \chi^s A_t^{1-\varrho}$$

Lagrangian:

$$\mathcal{L} = \left\{ \frac{\left(C_t - h\,C_{t-1}\right)^{1-\varrho}}{1-\varrho} - \chi_t^u \frac{\left(L_{ut}\right)^{1+\varphi}}{1+\varphi} - \chi_t^s \frac{\left(L_{st}\right)^{1+\varphi}}{1+\varphi} \right\} -$$
$$-\lambda_t \left\{ P_t C_t + D_t - D_{t-1} - W_{ut} L_{ut} - W_{st} L_{st} - DIV_t + P_t H_t + P_t T_t - \left(R_{t-1}^d - 1\right) D_{t-1} \right\}$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial C_t}: \qquad \left(C_t - h\,C_{t-1}\right)^{-\varrho} - \beta h \left(C_{t+1} - h\,C_t\right)^{-\varrho} - \lambda_t P_t = 0$$

$$u_{ct} = \left(C_t - h\,C_{t-1}\right)^{-\varrho} - \beta h \left(C_{t+1} - h\,C_t\right)^{-\varrho} \tag{A.1.1}$$

where: $\lambda_t P_t = u_{ct}$ (we call it the "real" MUC).

$$\frac{\partial \mathcal{L}}{\partial L_{ut}}: \qquad -\chi_t^u \left(L_{ut}\right)^{\varphi} - \lambda_t P_t \left(-\frac{W_{ut}}{P_t}\right) = 0$$

$$u_{ct}\, w_{ut} = \chi_t^u \left(L_{ut}\right)^{\varphi} \tag{A.1.2}$$

$$\frac{\partial \mathcal{L}}{\partial L_{st}}: \qquad -\chi_t^s \left(L_{st}\right)^{\varphi} - \lambda_t P_t \left(-\frac{W_{st}}{P_t}\right) = 0$$

$$u_{ct}\, w_{st} = \chi_t^s \left(L_{st}\right)^{\varphi} \tag{A.1.3}$$

where: w_t = real wage.

$$\frac{\partial \mathcal{L}}{\partial D_t}: \qquad -\lambda_t + \beta\lambda_{t+1} + \beta\lambda_{t+1}\left(R_t^d - 1\right) = 0$$

$$\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{P_{t+1}}{P_{t+1}}\frac{P_t}{P_t}R_t^d = 1$$

$$E_t\Lambda_{t,t+1}\pi_{t+1}^{-1}R_t^d = 1 \tag{A.1.4}$$

where the stochastic discount factor is:

$$\Lambda_{t,t+1} = \beta\frac{u_{ct+1}}{u_{ct}} \tag{A.1.5}$$

Banks

Net worth is:

$$Q_{Zt}F_{it}^z + Q_{Kt}F_{it}^k + Q_{Bt}B_{it} + RE_{it} = D_{it} + N_{it}$$

Net worth evolves according to:

$$N_{it} = \left(R_t^Z - R_{t-1}^d\right)Q_{Zt-1}F_{it-1}^z + \left(R_t^K - R_{t-1}^d\right)Q_{Kt-1}F_{it-1}^k + \\ + \left(R_t^B - R_{t-1}^d\right)Q_{Bt-1}B_{it-1} + + \left(R_{t-1}^{re} - R_{t-1}^d\right)RE_{it-1} + R_{t-1}^d N_{it-1}$$

Returns are:

$$R_t^Z = \frac{1 + \kappa_Z Q_{Zt}}{Q_{Zt-1}} \tag{A.1.6}$$

$$R_t^K = \frac{1 + \kappa_K Q_{Kt}}{Q_{Kt-1}} \tag{A.1.7}$$

$$R_t^B = \frac{1 + \kappa_B Q_{Bt}}{Q_{Bt-1}} \tag{A.1.8}$$

Banks' problem:

$$V_{it} = \max_{f_t^z, f_t^k, b_t, re_t} (1 - \sigma) E_t \sum_{j=1}^{\infty} \sigma^{j-1} \Lambda_{t,t+j} n_{it+j}$$

s.t. an incentive constraint:

$$V_{it} \geq \theta_t \left(Q_{Zt} f_{it}^z + \Delta_K Q_{Kt} f_{it}^k + \Delta_B Q_{Bt} b_{it} \right)$$

and a reserves constraint:

$$re_{it} \geq \varsigma_t d_{it}$$

where all small variables are real: es, $n_{i,t} = N_{it}/P_t$, is real net worth.

FOCs:

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^Z - R_t^d \right) = \frac{\lambda_t}{1 + \lambda_t} \theta_t \tag{A.1.9}$$

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^K - R_t^d \right) = \frac{\lambda_t}{1 + \lambda_t} \theta_t \Delta_K \tag{A.1.10}$$

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^B - R_t^d \right) = \frac{\lambda_t}{1 + \lambda_t} \theta_t \Delta_B \tag{A.1.11}$$

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_t^{re} - R_t^d \right) = -\frac{\omega_t}{1 + \lambda_t} \tag{A.1.12}$$

where:

$$\Omega_t = 1 - \sigma + \sigma \theta_t \phi_t \tag{A.1.13}$$

$$\phi_t = \frac{1+\lambda_t}{\theta_t} E_t \left[\Lambda_{t,t+1} \Omega_{t+1} \pi_{t+1}^{-1} \right] R_t^d - \frac{\omega_t re_t}{n_t \theta_t} \tag{A.1.14}$$

Labor Market

Wage Phillips Curves:

$$\max_{W_{lt}^*} E_t \sum_{j=0}^{\infty} \omega_w^j \left[-\chi_l \frac{\left(l_{ht+j}\right)^{1+\varphi}}{1+\varphi} + u_{ct+j} \frac{W_{lt}^* \Gamma_{wt,t+j}}{P_{t+j}} l_{ht+j} \right]$$

s.t. the demand for type l labour:

$$l_{ht} = \left(\frac{W_{hlt}}{W_{lt}} \right)^{\frac{\mu w}{1-\mu w}} l_t$$

where:

$$\Gamma_{wt,t+j} = \prod_{\tau=1}^{j} \left(\pi_{t+\tau-1}\right)^{\iota^w} \overline{\pi}^{1-\iota^w} g_t$$

FOCs:

$$E_t \left\{ \sum_{j=0}^{\infty} \omega_w^j \Lambda_{t,j} \left[\frac{W_{lt}^* \Gamma_{wt,t+j}}{P_{t+j}} - \mu_w \chi_{lt} \left(\frac{W_{lt}^* \Gamma_{wt,t+j}}{W_{lt+j}} \right)^{\frac{\varphi \mu w}{1-\mu w}} \frac{l_{t+j}^{\varphi}}{u_{ct+j}} \right] \left(\frac{W_{lt}^* \Gamma_{wt,t+j}}{W_{lt+j}} \right)^{\frac{\mu w}{1-\mu w}} l_{t+j} \right\} = 0$$

$$W_{lt} = \left[(1-\omega_w) \left(W_{lt}^*\right)^{1/(1-\mu_w)} + \omega_w \left(g_t \pi_{t-1}^{\iota_w} \overline{\pi}^{1-\iota_w} W_{lt-1} \right)^{1/(1-\mu_w)} \right]^{1-\mu_w}$$

In real terms:

$$E_t \{ \sum_{j=0}^{\infty} \omega_w^j \Lambda_{t,j} \left[\frac{w_{lt}^* \Gamma_{wt,t+j}}{\prod_{\tau=1}^{j} \pi_{t+\tau}} - \mu_w \chi_{lt} \left(\frac{w_{lt}^* \Gamma_{wt,t+j}}{w_{lt+j} \prod_{\tau=1}^{j} \pi_{t+\tau}} \right)^{\frac{\varphi \mu w}{1-\mu w}} \frac{X_{t+j}^{\varphi}}{u_{ct+j}} \right] *$$
$$* \left(\frac{w_{lt}^* \Gamma_{wt,t+j}}{w_{lt+j} \prod_{\tau=1}^{j} \pi_{t+\tau}} \right)^{\frac{\mu w}{1-\mu w}} X_{t+j} \} = 0$$

$$w_{lt} = \left[(1-\omega_w) \left(w^*_{lt} \right)^{1/(1-\mu_w)} + \omega_w \left(g_t \pi_{t-1}^{\iota_w} \overline{\pi}^{1-\iota_w} \frac{w_{lt-1}}{\pi_t} \right)^{1/(1-\mu_w)} \right]^{1-\mu_w}$$

Splitting by skilled and unskilled labour, wage PCs and laws of motion are:

$$E_t \{ \sum_{j=0}^{\infty} \omega_w^j \Lambda_{t,j} \left[\frac{w^*_{ut} \Gamma_{wt,t+j}}{\prod_{\tau=1}^{j} \pi_{t+\tau}} - \mu_w \chi_{ut} \left(\frac{w^*_{ut} \Gamma_{wt,t+j}}{w_{ut+j} \prod_{\tau=1}^{j} \pi_{t+\tau}} \right)^{\frac{\varphi \mu_w}{1-\mu_w}} \frac{L_{ut+j}^{\varphi}}{u_{ct+j}} \right] * \\ * \left(\frac{w^*_{ut} \Gamma_{wt,t+j}}{w_{ut+j} \prod_{\tau=1}^{j} \pi_{t+\tau}} \right)^{\frac{\mu_w}{1-\mu_w}} L_{ut+j} \} = 0 \tag{A.1.15}$$

$$w_{ut} = \left[(1-\omega_w) \left(w^*_{ut} \right)^{\frac{1}{1-\mu_w}} + \omega_w \left(g_t \pi_{t-1}^{\iota_w} \overline{\pi}^{1-\iota_w} \frac{w_{ut-1}}{\pi_t} \right)^{\frac{1}{1-\mu_w}} \right]^{1-\mu_w} \tag{A.1.16}$$

$$E_t \{ \sum_{j=0}^{\infty} \omega_w^j \Lambda_{t,j} \left[\frac{w^*_{st} \Gamma_{wt,t+j}}{\prod_{\tau=1}^{j} \pi_{t+\tau}} - \mu_w \chi_{st} \left(\frac{w^*_{st} \Gamma_{wt,t+j}}{w_{st+j} \prod_{\tau=1}^{j} \pi_{t+\tau}} \right)^{\frac{\varphi \mu_w}{1-\mu_w}} \frac{L_{st+j}^{\varphi}}{u_{ct+j}} \right] * \\ * \left(\frac{w^*_{st} \Gamma_{wt,t+j}}{w_{st+j} \prod_{\tau=1}^{j} \pi_{t+\tau}} \right)^{\frac{\mu_w}{1-\mu_w}} L_{st+j} \} = 0 \tag{A.1.17}$$

$$w_{st} = \left[(1-\omega_w) \left(w^*_{st} \right)^{\frac{1}{1-\mu_w}} + \omega_w \left(g_t \pi_{t-1}^{\iota_w} \overline{\pi}^{1-\iota_w} \frac{w_{st-1}}{\pi_t} \right)^{\frac{1}{1-\mu_w}} \right]^{1-\mu_w} \tag{A.1.18}$$

Capital Producers

For $i = \{k, z\}$, we have:

$$I_t^{Ni} = \left[1 - S \left(\frac{I_t^i}{I_{t-1}^i g_t} \right) \right] I_t^i \tag{A.1.19}$$

Dividends are:

$$DIV_{it} = P_t^i \left[1 - S \left(\frac{I_t^i}{I_{t-1}^i g_t} \right) \right] I_t^i - P_t I_t^i$$

in real terms:

$$div_{it} = p_t^i \left[1 - S \left(\frac{I_t^i}{I_{t-1}^i g_t} \right) \right] I_t^i - I_t^i$$

Capital producers' problem:

$$\max_{I_t^i} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left\{ p_{t+j}^i \left[1 - S\left(\frac{I_{t+j}^i}{I_{t+j-1}^i g_{t+j}} \right) \right] I_{t+j}^i - I_{t+j}^i \right\}$$

FOCs:

$$p_t^k \left[1 - S\left(\frac{I_t^k}{I_{t-1}^k g_t} \right) - S'\left(\frac{I_t^k}{I_{t-1}^k g_t} \right) \frac{I_t^k}{I_{t-1}^k g_t} \right] + E_t \Lambda_{t,t+1} p_{t+1}^k S'\left(\frac{I_{t+1}^k}{I_t^k g_{t+1}} \right) \left(\frac{I_{t+1}^k}{I_t^k g_{t+1}} \right)^2 = 1 \quad \text{(A.1.20)}$$

$$p_t^z \left[1 - S\left(\frac{I_t^z}{I_{t-1}^z g_t} \right) - S'\left(\frac{I_t^z}{I_{t-1}^z g_t} \right) \frac{I_t^z}{I_{t-1}^z g_t} \right] + E_t \Lambda_{t,t+1} p_{t+1}^z S'\left(\frac{I_{t+1}^z}{I_t^z g_{t+1}} \right) \left(\frac{I_{t+1}^z}{I_t^z g_{t+1}} \right)^2 = 1 \quad \text{(A.1.21)}$$

Final Good Producers

Price Phillips Curve:

$$\max_{P_{ft}^*} E_t \sum_{j=0}^{\infty} \omega^j \Lambda_{t,t+j} \left(\frac{P_{ft}^*}{P_{ft+j}} \Gamma_{t,t+j} - mc_{t+j} \right) Y_{ft+j}$$

s.t.

$$Y_{ft+j} = \left(\frac{P_{ft}^*}{P_{t+j}} \Gamma_{t,t+j} \right)^{\frac{\mu}{1-\mu}} Y_{t+j}$$

where:

$$\Gamma_{t,t+j} = \prod_{\tau=1}^{j} \left(1 + \pi_{t+\tau-1}\right)^{\iota^\pi} \overline{\pi}^{1-\iota^\pi}$$

Lagrangian:

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} \omega^j \Lambda_{t,t+j} \left(\frac{P_{ft}^*}{P_{ft+j}} \Gamma_{t,t+j} - mc_{t+j} \right) \left(\frac{P_{ft}^*}{P_{t+j}} \Gamma_{t,t+j} \right)^{\frac{\mu}{1-\mu}} Y_{t+j}$$

which can be rewritten as:

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} \omega^j \Lambda_{t,t+j} \left[\left(\frac{P_{ft}^*}{P_{ft+j}} \Gamma_{t,t+j} \right)^{\frac{1}{1-\mu}} - mc_{t+j} \left(\frac{P_{ft}^*}{P_{ft+j}} \Gamma_{t,t+j} \right)^{\frac{\mu}{1-\mu}} \right] Y_{t+j}$$

FOC:

$$\frac{\partial \mathcal{L}}{\partial P^*_{ft}}: \quad E_t \sum_{j=0}^{\infty} \omega^j \Lambda_{t,t+j} *$$

$$* \left[\frac{1}{1-\mu} \left(\frac{P^*_{ft}}{P_{ft+j}} \Gamma_{t,t+j} \right)^{\frac{\mu}{1-\mu}} \left(\frac{\Gamma_{t,t+j}}{P_{ft+j}} \right) - mc_{t,t+j} \frac{\mu}{1-\mu} \left(\frac{P^*_{ft}}{P_{ft+j}} \Gamma_{t,t+j} \right)^{-\frac{1}{1-\mu}} \left(\frac{\Gamma_{t,t+j}}{P_{ft+j}} \right) \right] Y_{t+j} = 0$$

Phillips curve is rewritten in terms of inflation rate π_t and optimal reset price $p^*_t \left(= P^*_t / P_{t-1}\right)$:

$$E_t \sum_{j=0}^{\infty} \omega_p^j \Lambda_{t,t+j} \left(\frac{P^*_{ft}}{P_{ft+j}} \Gamma_{t,t+j} \right)^{\frac{\mu}{1-\mu}} \left(\frac{P^*_{ft}}{P_{ft+j}} \Gamma_{t,t+j} - \mu mc_{t,t+j} \right) Y_{t+j} = 0$$

$$E_t \sum_{j=0}^{\infty} \omega_p^j \Lambda_{t,t+j} \left(\frac{P^*_t \Gamma_{t,t+j} P_{t-1}}{P_{t-1} P_{t+j}} \right)^{\frac{\mu}{1-\mu}} \left(\frac{P^*_t \Gamma_{t,t+j} P_{t-1}}{P_{t-1} P_{t+j}} - \mu mc_{t+j} \right) Y_{t+j} = 0$$

$$E_t \sum_{j=0}^{\infty} \omega_p^j \Lambda_{t,t+j} \left(\frac{p^*_t \Gamma_{t,t+j}}{\pi_{t+j}} \right)^{\frac{\mu}{1-\mu}} \left(\frac{p^*_t \Gamma_{t,t+j}}{\pi_{t+j}} - \mu mc_{t+j} \right) Y_{t+j} = 0 \tag{A.1.22}$$

Price Index is given by:

$$P_t = \left\{ \left(1-\omega_p\right) \left(P^*_t\right)^{\frac{1}{1-\mu}} + \omega_p \left(P_{t-1} \pi_{t-1}^{\iota^\pi} \pi^{1-\iota^\pi} \right)^{\frac{1}{1-\mu}} \right\}^{1-\mu}$$

$$\frac{P_t}{P_{t-1}} = \left\{ \left(1-\omega_p\right) \left(\frac{P^*_t}{P_{t-1}} \right)^{\frac{1}{1-\mu}} + \omega_p \left(\frac{P_{t-1}}{P_{t-1}} \pi_{t-1}^{\iota^\pi} \pi^{1-\iota^\pi} \right)^{\frac{1}{1-\mu}} \right\}^{1-\mu}$$

$$\pi_t = \left\{ \left(1-\omega_p\right) p_t^{*\frac{1}{1-\mu}} + \omega_p \left(\pi_{t-1}^{\iota^\pi} \overline{\pi}^{1-\iota^\pi} \right)^{\frac{1}{1-\mu}} \right\}^{1-\mu} \tag{A.1.23}$$

Wholesale Producers

Production function:

$$Y_{mt} = \epsilon_t^A \left(u_t K_t\right)^\alpha \left(L_{ut}\right)^{1-\alpha}$$

brought into aggregate terms following Anzoategui et al. (2019):

$$Y_t = A_t^{\vartheta-1}\epsilon_t^A \left(u_t K_t\right)^\alpha \left(L_{ut}\right)^{1-\alpha} \tag{A.1.24}$$

Capital law of motion:

$$K_{t+1} = I_t^N + \left(1-\delta\left(u_t\right)\right) K_t \tag{A.1.25}$$

Loan-in-advance constraint:

$$\psi^k P_t^k I_t^{Nk} \leq Q_{Kt} CK_{mt} = Q_{Kt}\left(F_{mt}^k - \kappa_K F_{mt-1}^k\right) \tag{A.1.26}$$

Dividends are:

$$DIV_{mt} = P_{mt}\epsilon_t^A \left(u_t K_t\right)^\alpha \left(L_{ut}\right)^{1-\alpha} - W_{ut}L_{ut} - P_t^k I_t^{Nk} - F_{mt-1}^k + Q_{Kt}\left(F_{mt}^k - \kappa_K F_{mt-1}^k\right)$$

in real terms:

$$div_{mt} = p_{mt}\epsilon_t^A \left(u_t K_t\right)^\alpha \left(L_{ut}\right)^{1-\alpha} - w_{ut}L_{ut} - p_t^k I_t^{Nk} + Q_{Kt}\left(\frac{F_{mt}^k}{P_t} - \kappa_K \frac{F_{mt-1}^k}{P_{t-1}}\pi_t^{-1}\right) - \frac{F_{mt-1}^k}{P_{t-1}}\pi_t^{-1}$$

Wholesale Producer Problem is: max A.1, s.t. A.1.26, A.1.25.

Lagrangian:

$$\begin{aligned}\mathcal{L}_{mt} = E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}\{ & p_{mt+j}\epsilon_{t+j}^A \left(u_{t+j}K_{t+j}\right)^\alpha \left(L_{t+j}^u\right)^{1-\alpha} - w_{t+j}^u L_{t+j}^u - p_{t+j}^k I_{t+j}^{Nk} + \\ & + Q_{Kt+j}\left(\frac{F_{mt+j}^k}{P_{t+j}} - \kappa_F \frac{F_{mt+j-1}^k}{P_{t+j-1}}\pi_{t+1}^{-1}\right) - \frac{F_{mt+j-1}^k}{P_{t+j-1}}\pi_{t+j}^{-1} + \\ & + v_{1t+j}\left[I_{t+j}^{Nk} + \left(1-\delta\left(u_{t+j}\right)\right)K_{t+j} - K_{t+j+1}\right] + \\ & + v_{2t+j}\left[Q_{Kt+j}\left(\frac{F_{mt+j}^k}{P_{t+j}} - \kappa_K \frac{F_{mt+j-1}^k}{P_{t+j-1}}\pi_{t+j}^{-1}\right) - \psi p_{t+j}^k I_{t+j}^{Nk}\right]\}\end{aligned}$$

FOCs:

$$\frac{\partial \mathcal{L}_{mt}}{\partial L_{ut}}: \qquad \mathcal{M} w_{ut} = (1-\alpha) p_{mt} \epsilon_t^A \left(u_t K_t\right)^{\alpha} \left(L_{ut}\right)^{-\alpha} \tag{A.1.27}$$

$$\frac{\partial \mathcal{L}_{mt}}{\partial I_t^{Nk}}: \qquad p_t^k \left(1 + v_{2t}\psi\right) = v_{1t}$$
$$v_{1t} = M_{1t} p_t^k$$

$$\frac{\partial \mathcal{L}_{mt}}{\partial u_t}: \qquad \mathcal{M} p_t^k M_{1t} \delta'\left(u_t\right) = \alpha p_{mt} \epsilon_t^A \left(u_t K_t\right)^{\alpha-1} \left(L_{ut}\right)^{1-\alpha} \tag{A.1.28}$$

$$\frac{\partial \mathcal{L}_{mt}}{\partial K_{t+1}}: \ \mathcal{M}\left[p_t^k M_{1t} - \left(1 - \delta\left(u_{t+1}\right)\right) p_{t+1}^k M_{1t+1}\right] = E_t \Lambda_{t,t+1} \left[\alpha p_{mt+1} \epsilon_{t+1}^A K_{t+1}^{\alpha-1} u_{t+1}^{\alpha} \left(L_{ut+1}\right)^{1-\alpha}\right] \tag{A.1.29}$$

$$\frac{\partial \mathcal{L}_{mt}}{\partial F_{mt}^k}: \qquad \mathcal{M}\left[Q_{Kt} M_{2t} - \pi_{t+1}^{-1} \kappa_K Q_{Kt+1} M_{2t+1}\right] = E_t \Lambda_{t,t+1} \pi_{t+1}^{-1} \tag{A.1.30}$$

where:

$$\frac{M_{1t} - 1}{M_{2t} - 1} = \psi \tag{A.1.31}$$

Innovators

$$\max_{F_{nt}^z, I_t^{Nz}, L_{st}} Q_t Z_t - p_t^z I_t^{Nz} - w_{st} L_{st} + Q_{Zt}\left(\frac{F_{nt}^z}{P_t} - \kappa_z \frac{F_{nt-1}^z}{P_{t-1}} \pi_t^{-1}\right) - \frac{F_{nt-1}^z}{P_{t-1}} \pi_t^{-1}$$

s.t.

$$Z_t = \left(I_t^{Nz}\right)^{\eta} \left(A_t L_{st}\right)^{1-\eta} \tag{A.1.32}$$

$$\psi^z p_t^z I_t^{Nz} \leq Q_{Zt}\left(\frac{F_{nt}^z}{P_t} - \kappa_Z \frac{F_{nt-1}^z}{P_{t-1}} \pi_t^{-1}\right) \tag{A.1.33}$$

Lagrangian:

$$\mathcal{L}_{nt} = E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \{ Q_t Z_t - p_t^z I_t^{Nz} - w_{st} L_{st} + Q_{Zt} \left(\frac{F_{nt}^z}{P_t} - \kappa_z \frac{F_{nt-1}^z}{P_{t-1}} \pi_t^{-1} \right) - \frac{F_{nt-1}^z}{P_{t-1}} \pi_t^{-1} +$$

$$+ v_{3t+j} \left[Z_t - \left(I_t^{Nz} \right)^{\eta} \left(A_t L_{st} \right)^{1-\eta} \right] +$$

$$+ v_{4t+j} \left[\psi^z p_t^z I_t^{Nz} - Q_{Zt} \left(\frac{F_{nt}^z}{P_t} - \kappa_Z \frac{F_{nt-1}^z}{P_{t-1}} \pi_t^{-1} \right) \right] \}$$

FOCs:

$$w_{st} = M_{3t} p_t^z \left(\frac{1-\eta}{\eta} \right) \frac{I_t^{Nz}}{L_{st}} \tag{A.1.34}$$

$$Q_{Zt} M_{4t} = E_t \Lambda_{t,t+1} \pi_{t+1}^{-1} \left[1 + Q_{Zt+1} \kappa_Z M_{4t+1} \right] \tag{A.1.35}$$

$$\frac{M_{3t} - 1}{M_{4t} - 1} = \psi^z \tag{A.1.36}$$

where: $M_{3t} = 1 + \psi^z v_{4t}$, and $M_{4t} = 1 + v_{4t}$. In addition: $v_{3t} = M_{3t} p_t^z / \left[\eta \left(I_t^{Nz} \right)^{\eta-1} \left(A_t L_{st} \right)^{1-\eta} \right]$.

$$A_{t+1} = \left(1 - \delta^A \right) \left(A_t + Z_t \right) \tag{A.1.37}$$

$$g_{t+1} = \frac{A_{t+1}}{A_t} = \left(1 - \delta^A \right) \left(1 + \frac{Z_t}{A_t} \right) \tag{A.1.38}$$

Fiscal Authority

$$P_t G_t + P_{t-1} \bar{b}_G = P_t T_t + P_t T_{cbt} + Q_{Bt} P_t \bar{b}_G \left(1 - \kappa_B \pi_t^{-1} \right)$$

In real terms:

$$G_t + \frac{\bar{b}_G}{\pi_t} = T_t + T_{cbt} + Q_{Bt} \bar{b}_G \left(1 - \kappa_B \pi_t^{-1} \right) \tag{A.1.39}$$

Central Bank

QE purchase:

$$Q_{Zt}F^{z}_{cbt} + Q_{Kt}F^{k}_{cbt} + Q_{Bt}B_{cbt} = RE_t$$

In real terms:

$$Q_{Zt}f^{z}_{cbt} + Q_{Kt}f^{k}_{cbt} + Q_{Bt}b_{cbt} = re_t \tag{A.1.40}$$

Remittances from the Central Bank to the Fiscal Authority:

$$\begin{aligned} T_{cbt} = {} & (1+\kappa_Z Q_{Zt})\,\pi_t^{-1} f^{z}_{cbt-1} + (1+\kappa_K Q_{Kt})\,\pi_t^{-1} f^{k}_{cbt-1} + \\ & + (1+\kappa Q_{Bt})\,\pi_t^{-1} b_{cbt-1} - R^{re}_{t-1}\pi_t^{-1} re_{t-1} \end{aligned} \tag{A.1.41}$$

Monetary Policy

$$\ln R^{TR}_t = \rho_r \ln R^{TR}_{t-1} + (1-\rho_r)\left[\ln \overline{R}^{TR} + \phi_\pi\left(\ln \pi_t - \ln \overline{\pi}\right) + \phi_y\left(\ln mc_t - \ln \overline{mc}\right)\right] + s_r \varepsilon_{rt} \tag{A.1.42}$$

Conventional policy in normal times:

$$R^d_t = R^{re}_t \tag{A.1.43}$$

$$R^{re}_t = R^{TR}_t \tag{A.1.44}$$

Conventional policy in ZLB times:

$$R^d_t = R^{re}_t \tag{A.1.45}$$

$$R^{re}_t = \max\{1,\, R^{TR}_t\} \tag{A.1.46}$$

NIRP times:

$$R^d_t = \max\{1,\, R^{re}_t\} \quad \text{with} \quad R^{re}_t = R^{TR}_t \tag{A.1.47}$$

Exogenous bond holdings:

$$f^{z}_{cbt} = (1-\rho_z)\, f^{z}_{cb} + \rho_z f^{z}_{cbt-1} + s_z \varepsilon_{zt} \tag{A.1.48}$$

$$f^{k}_{cbt} = (1-\rho_k)\, f^{k}_{cb} + \rho_k f^{k}_{cbt-1} + s_k \varepsilon_{kt} \tag{A.1.49}$$

$$b_{cbt} = (1-\rho_b)\, b_{cb} + \rho_b b_{cbt-1} + s_b \varepsilon_{bt} \tag{A.1.50}$$

Aggregation, Market Clearing & Equilibrium

Auxiliary definitions are:

$$\delta(u_t) = \delta_0 + \delta_1 (u_t - 1) + \frac{\delta_2}{2} (u_t - 1)^2$$

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

Exogenous processes for TFP, public spending and liquidity:

$$\ln \epsilon^{A}_{t} = \rho_A \ln \epsilon^{A}_{t-1} + s_A \varepsilon_{At} \tag{A.1.51}$$

$$\ln G_t = (1-\rho_G) \ln G + \rho_G \ln G_{t-1} + s_G \varepsilon_{Gt} \tag{A.1.52}$$

$$\ln \theta_t = (1-\rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + s_\theta \varepsilon_{\theta t} \tag{A.1.53}$$

Bond market clearing conditions:

$$f^{z}_{nt} = f^{z}_{t} + f^{z}_{cbt} \tag{A.1.54}$$

$$f^{k}_{mt} = f^{k}_{t} + f^{k}_{cbt} \tag{A.1.55}$$

$$\overline{b}_G = b_t + b_{cbt} \tag{A.1.56}$$

Banks' balance sheets:

$$Q_{Zt} f^{z}_{t} + Q_{Kt} f^{k}_{t} + Q_{Bt} b_t + re_t = d_t + n_t \tag{A.1.57}$$

Aggregate net worth dynamics:

$$
\begin{aligned}
n_t = \sigma\pi_t^{-1}[&\left(R_t^Z - R_{t-1}^d\right)Q_{Zt-1}f_{t-1}^z + \left(R_t^K - R_{t-1}^d\right)Q_{Kt-1}f_{t-1}^k + \\
&+\left(R_t^B - R_{t-1}^d\right)Q_{Bt-1}b_{t-1} + \left(R_{t-1}^{re} - R_{t-1}^d\right)re_{t-1} + R_{t-1}^d n_{t-1}] + H_t
\end{aligned}
\tag{A.1.58}
$$

Leverage:

$$
Q_{Zt}f_t^z + \Delta_K Q_{Kt}f_t^k + \Delta_B Q_{Bt}b_t \leq \phi_t n_t \tag{A.1.59}
$$

Aggregate labour:

$$
L_t = L_{st} + L_{ut} \tag{A.1.60}
$$

Aggregate resource constraint:

$$
Y_t = C_t + I_t^k + I_t^z + G_t \tag{A.1.61}
$$

A.2 Stationarized Model

Tilde variable denotes the detrended version of the same variable. Following Queralto (2020), we define: $\widetilde{X}_t = \frac{X_t}{A_t}$, while marginal utility of consumption is detrended by $A_t^{-\varrho}\left(\widetilde{u}_{ct} = \frac{u_{ct}}{A_t^{-\varrho}}\right)$.

Households

$$\widetilde{u}_{ct} = \left(C_t - h\frac{\widetilde{C}_{t-1}}{g_t}\right)^{-\varrho} - \beta h\left(\widetilde{C}_{t+1}g_{t+1} - h\widetilde{C}_t\right)^{-\varrho} \tag{A.2.1}$$

$$\widetilde{u}_{ct}\widetilde{w}_t^u = \chi^u\left(L_{ut}\right)^{\varphi} \tag{A.2.2}$$

$$\widetilde{u}_{ct}\widetilde{w}_t^s = \chi^s\left(L_{st}\right)^{\varphi} \tag{A.2.3}$$

$$\begin{aligned}
\Lambda_{t,t+1} &= \beta\frac{u_{ct+1}}{u_{ct}}\left(\frac{A_t}{A_t}\right)^{-\sigma}\left(\frac{A_{t+1}}{A_{t+1}}\right)^{-\sigma} \\
\Lambda_{t,t+1} &= \beta\frac{\widetilde{u}_{ct+1}}{\widetilde{u}_{ct}}\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma} \\
\Lambda_{t,t+1} &= \frac{\widetilde{\Lambda}_{t,t+1}}{g_{t+1}{}^{\sigma}}
\end{aligned} \tag{A.2.4}$$

$$E_t\frac{\widetilde{\Lambda}_{t,t+1}}{g_{t+1}{}^{\sigma}}\pi_{t+1}^{-1}R_t^d = 1 \tag{A.2.5}$$

Bankers

Returns are stationary:

$$R_t^Z = \frac{1+\kappa_Z Q_{Zt}}{Q_{Zt-1}} \tag{A.2.6}$$

$$R_t^K = \frac{1+\kappa_K Q_{Kt}}{Q_{Kt-1}} \tag{A.2.7}$$

$$R_t^B = \frac{1+\kappa_B Q_{Bt}}{Q_{Bt-1}} \tag{A.2.8}$$

$$E_t \frac{\widetilde{\Lambda}_{t,t+1}}{{g_{t+1}}^{\sigma}} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^{Z} - R_t^d \right) = \frac{\lambda_t}{1+\lambda_t} \theta_t \tag{A.2.9}$$

$$E_t \frac{\widetilde{\Lambda}_{t,t+1}}{{g_{t+1}}^{\sigma}} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^{K} - R_t^d \right) = \frac{\lambda_t}{1+\lambda_t} \theta_t \Delta_K \tag{A.2.10}$$

$$E_t \frac{\widetilde{\Lambda}_{t,t+1}}{{g_{t+1}}^{\sigma}} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_{t+1}^{B} - R_t^d \right) = \frac{\lambda_t}{1+\lambda_t} \theta_t \Delta_B \tag{A.2.11}$$

$$E_t \frac{\widetilde{\Lambda}_{t,t+1}}{{g_{t+1}}^{\sigma}} \Omega_{t+1} \pi_{t+1}^{-1} \left(R_t^{re} - R_t^d \right) = -\frac{\omega_t}{1+\lambda_t} \tag{A.2.12}$$

$$\Omega_t = 1 - \sigma + \sigma \theta_t \phi_t \tag{A.2.13}$$

$$\phi_t = \frac{1+\lambda_t}{\theta_t} E_t \left[\frac{\widetilde{\Lambda}_{t,t+1}}{{g_{t+1}}^{\sigma}} \Omega_{t+1} \pi_{t+1}^{-1} \right] R_t^d - \frac{\omega_t \widetilde{re}_t}{\widetilde{n}_t \theta_t} \tag{A.2.14}$$

Capital Producers

$$\widetilde{I}_t^{Nk} = \left[1 - S \left(\frac{\widetilde{I}_t^k}{\widetilde{I}_{t-1}^k} \right) \right] \widetilde{I}_t^k \tag{A.2.15}$$

$$\widetilde{I}_t^{Nz} = \left[1 - S \left(\frac{\widetilde{I}_t^z}{\widetilde{I}_{t-1}^z} \right) \right] \widetilde{I}_t^z \tag{A.2.16}$$

$$p_t^k \left[1 - S \left(\frac{\widetilde{I}_t^k}{\widetilde{I}_{t-1}^k} \right) - S' \left(\frac{\widetilde{I}_t^k}{\widetilde{I}_{t-1}^k} \right) \frac{\widetilde{I}_t^k}{\widetilde{I}_{t-1}^k} \right] + E_t \frac{\widetilde{\Lambda}_{t,t+1}}{{g_{t+1}}^{\sigma}} p_{t+1}^k S' \left(\frac{\widetilde{I}_{t+1}^k}{\widetilde{I}_t^k} \right) \left(\frac{\widetilde{I}_{t+1}^k}{\widetilde{I}_t^k} \right)^2 = 1 \tag{A.2.17}$$

$$p_t^z \left[1 - S \left(\frac{\widetilde{I}_t^z}{\widetilde{I}_{t-1}^z} \right) - S' \left(\frac{\widetilde{I}_t^z}{\widetilde{I}_{t-1}^z} \right) \frac{\widetilde{I}_t^z}{\widetilde{I}_{t-1}^z} \right] + E_t \frac{\widetilde{\Lambda}_{t,t+1}}{{g_{t+1}}^{\sigma}} p_{t+1}^z S' \left(\frac{\widetilde{I}_{t+1}^z}{\widetilde{I}_t^z} \right) \left(\frac{\widetilde{I}_{t+1}^z}{\widetilde{I}_t^z} \right)^2 = 1 \tag{A.2.18}$$

Wholesale Producers

Applying the methodology in Anzoategui et al. (2019), conditions are expressed as a function of marginal costs.

Definitions:

$$L_t = A_t L_{umt}$$

$$K_t = A_t K_{mt}$$

$$u_t = u_{mt}$$

$$mc_t = \frac{p_{mt}}{A_t^{\vartheta-1}}$$

Applying the definitions above:

$$\begin{aligned}
Y_t &= \left(\int_0^{A_t} X_{mt}^{\frac{1}{\vartheta}} dm \right)^{\vartheta} \\
&= A_t^{\vartheta} \left\{ \varepsilon_t^A \left(u_{mt} K_{mt} \right)^{\alpha} L_{mt}^{1-\alpha} \right\} \\
&= A_t^{\vartheta} \left\{ \varepsilon_t^A \left(u_t \frac{K_t}{A_t} \right)^{\alpha} \left(\frac{L_t}{A_t} \right)^{1-\alpha} \right\} \\
&= A_t^{\vartheta-1} \left\{ \varepsilon_t^A \left(u_t K_t \right)^{\alpha} L_t^{1-\alpha} \right\} \\
&= A_t^{\vartheta-1} X_t
\end{aligned}$$

Under the parametrization such that $(\frac{\vartheta-1}{1-\alpha}) = 0$:

$$\widetilde{Y}_t = \epsilon_t^A \left(u_t \widetilde{K}_t \right)^{\alpha} L_t^{1-\alpha} \tag{A.2.19}$$

$$g_{t+1} \widetilde{K}_{t+1} = \widetilde{I}_t^{Nk} + \left(1 - \delta\left(u_t\right)\right) \widetilde{K}_t \tag{A.2.20}$$

$$\psi^k p_t^k \widetilde{I}_t^{Nk} \leq Q_{Kt} \left(\frac{\widetilde{F}_{mt}^k}{P_t} - \kappa_K \frac{\widetilde{F}_{mt-1}^k}{P_{t-1}} \frac{\pi_t^{-1}}{g_t} \right) \tag{A.2.21}$$

$$\mathcal{M} \widetilde{w}_t^u = (1-\alpha) mc_t \frac{\widetilde{Y}_t}{L_{ut}} \tag{A.2.22}$$

$$\mathcal{M} M_{1t} p_t^k \delta'\left(u_t\right) \widetilde{K}_t = \alpha\, mc_t \frac{\widetilde{Y}_t}{u_t} \tag{A.2.23}$$

$$\mathcal{M} \left[M_{1t} p_t^k - E_t \frac{\widetilde{\Lambda}_{t,t+1}}{g_{t+1}{}^{\sigma}} \left(1 - \delta\left(u_{t+1}\right)\right) M_{1t+1} p_{t+1}^k \right] = E_t \frac{\widetilde{\Lambda}_{t,t+1}}{g_{t+1}{}^{\sigma}} \alpha\, mc_{t+1} \frac{\widetilde{Y}_{t+1}}{\widetilde{K}_{t+1}} \tag{A.2.24}$$

$$\mathcal{M}\left[Q_{Kt}M_{2t} - E_t\frac{\widetilde{\Lambda}_{t,t+1}}{g_{t+1}{}^{\sigma}}\pi_{t+1}^{-1}\kappa_K Q_{Kt+1}M_{2t+1}\right] = E_t\frac{\widetilde{\Lambda}_{t,t+1}}{g_{t+1}{}^{\sigma}}\pi_{t+1}^{-1} \tag{A.2.25}$$

$$\frac{M_{1t}-1}{M_{2t}-1} = \psi \tag{A.2.26}$$

Final Good Producers

$$E_t\sum_{j=0}^{\infty}\omega_p^j\frac{\widetilde{\Lambda}_{t,t+j}}{g_{t+j}{}^{\sigma}}\left(\frac{p_t^*\Gamma_{t,t+j}}{\pi_{t+j}}\right)^{\frac{\mu}{1-\mu}}\left(\frac{p_t^*\Gamma_{t,t+j}}{\pi_{t+j}} - \mu mc_{t+j}\right)\widetilde{Y}_{t+j} = 0$$

p_t^* is then expressed as the ratio of terms F_{pt}, Z_{pt}:

$$p_t^* = \frac{F_{pt}}{Z_{pt}} \tag{A.2.27}$$

where:

$$F_{pt} = E_t\sum_{j=0}^{\infty}\omega_p^j\frac{\widetilde{\Lambda}_{t,t+j}}{g_{t+j}{}^{\sigma}}mc_{t+j}\mu\left(\frac{\Gamma_{t,t+j}}{\pi_{t+j}}\right)^{\frac{\mu}{1-\mu}}\widetilde{Y}_{t+j}$$

$$Z_{pt} = E_t\sum_{j=0}^{\infty}\omega_p^j\frac{\widetilde{\Lambda}_{t,t+j}}{g_{t+j}{}^{\sigma}}\left(\frac{\Gamma_{t,t+j}}{\pi_{t+j}}\right)^{\frac{1}{1-\mu}}\widetilde{Y}_{t+j}$$

Recursively, the same factors are defined as:

$$F_{pt} = mc_t\mu\left(\frac{\Gamma_t}{\pi_t}\right)^{\frac{\mu}{1-\mu}}\widetilde{Y}_t + \omega_p\frac{\widetilde{\Lambda}_{t,t+1}}{g_{t+1}{}^{\sigma}}E_tF_{pt+1} \tag{A.2.28}$$

$$Z_{pt} = \left(\frac{\Gamma_t}{\pi_t}\right)^{\frac{1}{1-\mu}}\widetilde{Y}_t + \omega_p\frac{\widetilde{\Lambda}_{t,t+1}}{g_{t+1}{}^{\sigma}}E_tZ_{pt+1} \tag{A.2.29}$$

Labour Market

$$E_t\{\sum_{j=0}^{\infty}\omega_w^j\widetilde{\Lambda}_{t,t+j}\left[\frac{\widetilde{w}_{ut}^*\Gamma_{wt,t+j}}{\prod_{k=1}^{j}\pi_{t+k}g_{t+j}{}^{\sigma}} - \mu_w\chi_u\left(\frac{\widetilde{w}_{ut}^*\Gamma_{wt,t+j}}{\widetilde{w}_{ut+j}\prod_{k=1}^{j}\pi_{t+k}g_{t+j}}\right)^{\frac{\varphi\mu_w}{1-\mu_w}}\frac{L_{ut+j}^{\varphi}}{\widetilde{u}_{ct+j}}\right]*$$

$$*\left(\frac{\widetilde{w}_{ut}^*\Gamma_{wt,t+j}}{\widetilde{w}_{ut+j}\prod_{k=1}^{j}\pi_{t+k}g_{t+j}}\right)^{\frac{\mu_w}{1-\mu_w}}L_{ut+j}\} = 0$$

$$E_t\{\sum_{j=0}^{\infty}\omega_w^j\widetilde{\Lambda}_{t,t+j}\left[\frac{\widetilde{w}_{st}^*\Gamma_{wt,t+j}}{\prod_{k=1}^{j}\pi_{t+k}g_{t+j}{}^{\sigma}}-\mu_w\chi_s\left(\frac{\widetilde{w}_{st}^*\Gamma_{wt,t+j}}{\widetilde{w}_{st+j}\prod_{k=1}^{j}\pi_{t+k}g_{t+j}}\right)^{\frac{\varphi\mu_w}{1-\mu_w}}\frac{L_{st+j}^{\varphi}}{\widetilde{u}_{ct+j}}\right]*$$
$$*\left(\frac{\widetilde{w}_{st}^*\Gamma_{wt,t+j}}{\widetilde{w}_{st+j}\prod_{k=1}^{j}\pi_{t+k}g_{t+j}}\right)^{\frac{\mu_w}{1-\mu_w}}L_{st+j}\}=0$$

Following the procedure for the price PC above, I obtain:

$$\widetilde{w}_{ut}^*=\left(\frac{F_{ut}}{Z_{ut}}\right)^{\frac{1-\mu-\varphi\mu}{1-\mu}} \tag{A.2.30}$$

where:

$$F_{ut}=\left[\mu\chi_u\left(\frac{\Gamma_t}{\widetilde{w}_{ut}\pi_t g_t}\right)^{\frac{\mu(1+\varphi)}{1-\mu}}\frac{L_{ut}^{\varphi+1}}{\widetilde{u}_{ct}}\right]^{\frac{1-\mu}{\mu+\varphi\mu-1}}+\omega_w\widetilde{\Lambda}_{t,t+1}E_tZ_{ut+1} \tag{A.2.31}$$

$$Z_{ut}=\left[\left(\frac{1}{\widetilde{w}_{ut}}\right)^{\frac{\mu}{1-\mu}}\left(\frac{\Gamma_t}{\pi_t g_t{}^{\sigma}}\right)^{\frac{1}{1-\mu}}L_{ut}\right]^{\frac{1-\mu}{\mu+\varphi\mu-1}}+\omega_w\widetilde{\Lambda}_{t,t+1}E_tF_{ut+1} \tag{A.2.32}$$

$$\widetilde{w}_{st}^*=\left(\frac{F_{st}}{Z_{st}}\right)^{\frac{1-\mu}{\mu+\varphi\mu-1}} \tag{A.2.33}$$

where:

$$F_{st}=\left[\left(\frac{1}{\widetilde{w}_{st}}\right)^{\frac{\mu}{1-\mu}}\left(\frac{\Gamma_t}{\pi_t g_t{}^{\sigma}}\right)^{\frac{1}{1-\mu}}L_{st}\right]^{\frac{1-\mu}{\mu+\varphi\mu-1}}+\omega_w\widetilde{\Lambda}_{t,t+1}F_{st+1} \tag{A.2.34}$$

$$Z_{st}=\left[\mu\chi_s\left(\frac{1}{\widetilde{w}_{st}}\right)^{\frac{\mu(1+\varphi)}{1-\mu}}\left(\frac{\Gamma_t}{\pi_t g_t}\right)^{\frac{\mu(1+\varphi)}{1-\mu}}\frac{L_{st}^{\varphi+1}}{\widetilde{u}_{ct}}\right]^{\frac{1-\mu}{\mu+\varphi\mu-1}}+\omega_w\widetilde{\Lambda}_{t,t+1}Z_{st+1} \tag{A.2.35}$$

Innovators

$$\widetilde{Z}_t=\left(\widetilde{I}_t^{Nz}\right)^{\eta}(L_{st})^{1-\eta} \tag{A.2.36}$$

$$\psi^z p_t^z\widetilde{I}_t^{Nz}\leq Q_{Zt}\left(\frac{\widetilde{F}_{nt}^z}{P_t}-\kappa_Z\frac{\widetilde{F}_{nt-1}^z}{P_{t-1}}\frac{\pi_t^{-1}}{g_t}\right) \tag{A.2.37}$$

$$\widetilde{w}_{st} = M_{3t} p_t^z \left(\frac{1-\eta}{\eta}\right) \frac{\widetilde{I}_t^{Nz}}{L_{st}} \tag{A.2.38}$$

$$Q_{Zt} M_{4t} = E_t \frac{\widetilde{\Lambda}_{t,t+1}}{{g_{t+1}}^{\sigma}} \pi_{t+1}^{-1} \left[1 + Q_{Zt+1} \kappa_Z M_{4t+1}\right] \tag{A.2.39}$$

$$\frac{M_{3t} - 1}{M_{4t} - 1} = \psi^z \tag{A.2.40}$$

$$g_{t+1} = \frac{A_{t+1}}{A_t} = \left(1 - \delta^A\right)\left(1 + \widetilde{Z}_t\right) \tag{A.2.41}$$

Fiscal Authority

$$G_t + \frac{\overline{b}_G}{\pi_t} = \widetilde{T}_t + \widetilde{T}_{cbt} + Q_{Bt} \overline{b}_G \left(1 - \kappa_B \pi_t^{-1}\right) \tag{A.2.42}$$

Central Bank

$$Q_{Zt} f_{cbt}^z + Q_{Kt} f_{cbt}^k + Q_{Bt} b_{cbt} = \widetilde{re}_t \tag{A.2.43}$$

$$g_t \widetilde{T}_{cbt} = \left(1 + \kappa_Z Q_{Zt}\right) \pi_t^{-1} f_{cbt-1}^z + \left(1 + \kappa_K Q_{Kt}\right) \pi_t^{-1} f_{cbt-1}^k + \left(1 + \kappa Q_{Bt}\right) \pi_t^{-1} b_{cbt-1} - R_{t-1}^{re} \pi_t^{-1} \widetilde{re}_{t-1} \tag{A.2.44}$$

Monetary Policy

$$\ln R_t^{TR} = \rho_r \ln R_{t-1}^{TR} + (1 - \rho_r)\left[\ln \overline{R}^{TR} + \phi_\pi \left(\ln \pi_t - \ln \overline{\pi}\right) + \phi_y \left(\ln mc_t - \ln \overline{mc}\right)\right] + s_r \varepsilon_{rt} \tag{A.2.45}$$

$$R_t^d = R_t^{re} \tag{A.2.46}$$

$$R_t^{re} = R_t^{TR} \tag{A.2.47}$$

$$f_{cbt}^z = (1 - \rho_z) f_{cb}^z + \rho_z f_{cbt-1}^z + s_z \varepsilon_{zt} \tag{A.2.48}$$

$$f_{cbt}^{k} = (1-\rho_k)\, f_{cb}^{k} + \rho_k f_{cbt-1}^{k} + s_k \varepsilon_{kt} \tag{A.2.49}$$

$$b_{cbt} = (1-\rho_b)\, b_{cb} + \rho_b b_{cbt-1} + s_b \varepsilon_{bt} \tag{A.2.50}$$

Market Clearing

$$\ln \epsilon_t^A = \rho_A \ln \epsilon_{t-1}^A + s_A \varepsilon_{At} \tag{A.2.51}$$

$$\ln G_t = (1-\rho_G) \ln G + \rho_G \ln G_{t-1} + s_G \varepsilon_{Gt} \tag{A.2.52}$$

$$\ln \theta_t = (1-\rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + s_\theta \varepsilon_{\theta t} \tag{A.2.53}$$

$$\widetilde{f}_{nt}^{z} = \widetilde{f}_{t}^{z} + f_{cbt}^{z} \tag{A.2.54}$$

$$\widetilde{f}_{mt}^{k} = \widetilde{f}_{t}^{k} + f_{cbt}^{k} \tag{A.2.55}$$

$$\overline{b}_G = \widetilde{b}_t + b_{cbt} \tag{A.2.56}$$

$$L_t = L_{st} + L_{ut} \tag{A.2.57}$$

$$(\pi_t)^{\frac{1}{1-\mu}} = \left(1-\omega_p\right)\left(p_t^*\right)^{\frac{1}{1-\mu}} + \omega_p \left(\pi_{t-1}^{\iota^\pi} \overline{\pi}^{1-\iota^\pi}\right)^{\frac{1}{1-\mu}} \tag{A.2.58}$$

$$(\widetilde{w}_{ut})^{\frac{1}{1-\mu_w}} = (1-\omega_w)\left(\widetilde{w}_{ut}^*\right)^{\frac{1}{1-\mu_w}} + \omega_w \left(\pi_{t-1}^{\iota_w} \overline{\pi}^{1-\iota_w} \frac{\widetilde{w}_{ut-1}}{\pi_t}\right)^{\frac{1}{1-\mu_w}} \tag{A.2.59}$$

$$(\widetilde{w}_{st})^{\frac{1}{1-\mu_w}} = (1-\omega_w)\left(\widetilde{w}_{st}^*\right)^{\frac{1}{1-\mu_w}} + \omega_w \left(\pi_{t-1}^{\iota_w} \overline{\pi}^{1-\iota_w} \frac{\widetilde{w}_{st-1}}{\pi_t}\right)^{\frac{1}{1-\mu_w}} \tag{A.2.60}$$

$$Q_{Zt} \widetilde{f}_t^{z} + Q_{Kt} \widetilde{f}_t^{k} + Q_{Bt} \widetilde{b}_t + \widetilde{re}_t = \widetilde{d}_t + \widetilde{n}_t \tag{A.2.61}$$

$$\widetilde{n}_t = \frac{\sigma}{g_t \pi_t}[\left(R_t^Z - R_{t-1}^d\right) Q_{Zt-1} \widetilde{f}_{t-1}^z + \left(R_t^K - R_{t-1}^d\right) Q_{Kt-1} \widetilde{f}_{t-1}^k + \\ + \left(R_t^B - R_{t-1}^d\right) Q_{Bt-1} \widetilde{b}_{t-1} + \left(R_{t-1}^{re} - R_{t-1}^d\right) \widetilde{re}_{t-1} + R_{t-1}^d \widetilde{n}_{t-1}] + \widetilde{H}_t \tag{A.2.62}$$

$$Q_{Zt} \widetilde{z}_t + \Delta_K Q_{Kt} \widetilde{f}_t^k + \Delta_B Q_{Bt} \widetilde{b}_t \leq \phi_t \widetilde{n}_t \tag{A.2.63}$$

$$\widetilde{Y}_t = \widetilde{C}_t + \widetilde{I}_t^z + \widetilde{I}_t^k + G_t \tag{A.2.64}$$

A.3 Complete Calibration

Parameter	Definition	Value / Target *	Source
Households			
β	Discount Factor	0.995	Literature
χ^u	Unskilled Labour Disutility	* L = 0.33	Literature
χ^s	Skilled Labour Disutility	* $\overline{L_s}/L = 12.8\%$	*NSF*
h	Habit Formation	0.700	Sims & Wu (2021)
φ	Inverse of Frisch Elasticity	1	"
Bankers			
σ	Survival Rate	0.950	Sims & Wu (2021)
θ	Recoverability parameter	* Leverage = 4	"
κ_B	Government Bond Duration	1-40^{-1}	Sims & Wu (2021)
κ_K	Private Bond Duration	1-39^{-1}	"
κ_Z	Innovation Bond Duration	1-38^{-1}	"
Δ_B	Government Bond Recoverability	* $(RK - Rd) = 0.0075$	Bonciani et al. (2023)
Δ_K	Private Bond Recoverability	* $(RZ - Rd) = 0.0115$	"
Non-financial firms			
α	Capital Share	0.330	Literature
ϑ	Desired Markup on Interm. Good X	1.670	Queralto (2020)
μ	Markup on Final Good Y	1.100	Anzoategui et al. (2019)
$\mathcal{M}$	Effective Markup on Interm. Good X	1.180	"
μ_w	Markup on Wages	0.150	"
ω_p	Calvo Price Adjustment	0.750	Sims & Wu (2021)
ι^π	Price Indexation	0.000	"
ω_w	Calvo Wage Adjustment	0.750	"
ι^w	Wage Indexation	0.000	"
δ_0	Capital Depreciation (SS)	0.025	"
δ_1	Utilization, Linear Term	* u = 1	"
δ_2	Utilization, Squared Term	0.010	"
κ_I	Investment Adjustment Cost	2.000	"
ψ^k	Fraction of Investment from Debt	0.800	"
Technology Sectors			
ψ^z	Fraction of Investment from Debt	0.900	
δ^A	Technology Depreciation (SS)	0.030	Bonciani et al. (2023)
η	Capital Share in Innovation	0.190	Queralto (2020)
Central Bank			
$\overline{\pi}$	Inflation Target	1.000	Sims & Wu (2021)
ϕ^π	Inflation Reaction Coefficient	1.500	"
ϕ^y	Output Reaction Coefficient	0.250	"
ρ_R	Taylor Rule Smoothing	0.800	"
b_{cb}	CB Treasury Holdings (SS)	0.06	"
f^k_{cb}	CB Private Bond Holdings (SS)	0	"
$\underline{f}^z_{cb}$	CB Innovation Bond Holdings (SS)	0	"
b_G	Government Debt (SS)	0.410	"
G	Government Spending (SS)	* Ratio G/Y = 20%	"
$\overline{r}$	SS Nominal Interest Rate	* Y Growth = 1.8%	*FRED*
$\overline{mc}$	SS Marginal Costs	* 1/Markup (SS)	
Shock Processes			
ρ_θ	Liquidity Shock Persistence	0.980	Sims & Wu (2021)
ρ_A	TFP Shock Persistence	0.950	"
ρ_G	Government Spending Persistence	0.950	"
ρ_b	Treasury Holdings Persistence	0.980	"
ρ_k	Private Bonds Persistence	0.800	"
ρ_z	Innovation Bonds Persistence		"
s_r	Monetary Shock SD	0.0025	Sims & Wu (2021)
s_θ	Liquidity Shock SD	0.040	"
s_A	TFP Shock SD	0.0065	"
s_G	Gov. Spending Shock SD	0.010	"
s_b	Gov. Bond SD	0.000	"
s_k	Private Bond SD	0.0025	"
s_z	Innovation Bond SD		"

Table 3: Calibrated Parameters